Sixth Workshop on Fourier Analysis and Related Fields

Conference Booklet

Pécs, 2017

Sixth Workshop on Fourier Analysis and Related Fields

The Alfréd Rényi Institute of Mathematics of the Hungarian Academy of Sciences and the Institute of Mathematics and Informatics, Faculty of Sciences, University of Pécs organizes the Sixth Workshop on Fourier Analysis and Related Fields. The workshop takes place in Pécs, Hungary between August 24 and August 31, 2017.

This workshop is a continuation of the series of Fourier workshops held at the Rényi Institute in 2005, 2007, 2009, 2013, 2015. The workshop focuses on Fourier analysis and its applications in various branches of mathematics, such as additive combinatorics, number theory and mathematical analysis.

The organizing committee is: Tímea Eisner, Marcel Gaál, Ágota G. Horváth, Balázs Király, Béla Nagy, Margit Pap (co-chair), Viktor Rébay, Szilárd Révész (chairman), Ilona Simon (secretary), Tamás Titkos, László Tóth, Dániel Virosztek.

The venue will be in the Hungarian Academy of Sciences local Headquarters, Pécs. The address is Jurisics Miklós street 44, Pécs. See Section 1 for map and more directions to the site.

Welcome to Pécs!

It is our honor to welcome you to the Sixth Workshop on Fourier Analysis and Related Fields. We are glad to host this event in year 2017, when the University of Pécs is celebrating the 650th jubilee of the foundation as the first university in Hungary. Faculty of Sciences is also celebrating its 25th anniversary of foundation.

Hope that you will enjoy the scientific atmosphere of the conference as well as the historical and Mediterranean atmosphere of Pécs.

Szilárd Révész, chair,
Margit Pap, co-chair

The workshop gratefully acknowledges the financial support of

- Alfréd Rényi Institute of Mathematics of the Hungarian Academy of Sciences
  `http://www.renyi.hu/`
- University of Pécs, Faculty of Sciences, Institute of Informatics
  and Mathematics `http://www.ttk.pte.hu/en`
- European Social Funds EFOP-3.4.3-16-2016-00005 and EFOP-3.6.1.-16-2016-00004
  `http://pii.pte.hu/content/efop-343-16-2016-00005`.
- Akadémiai Kiadó (Publishing House of the Hungarian Academy of Sciences)
  `https://akademiai.hu`

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Schedule

August 24, Thursday. Venue: PAB

19:00 Registration

20:00 Welcome Party

August 25, Friday. Venue: PAB

09:15 Opening

09:30	H.G. Feichtinger: On the abundance of Fourier standard spaces
10:15	V. Ivanov: Some extremal problems for Fourier transform on hyperboloid

11:00-11:30 Coffee break

11:30	T. Hilberdink: Matrices with multiplicative entries
12:15	F. Weisz: Convergence of rectangular summability and Lebesgue points of higher dimensional Fourier transforms
12:40	Yu. Kolomoitsev: Approximation by multivariate Kantorovich–Kotelnikov sampling operators

13:10-14:30 Lunch break

14:30	Y. Rouba: Rational Fourier-Chebyshev series of some elementary functions	G. Bhowmik: Goldbach representations with congruences
15:15	I. Blahota: Approximation by θ-means of Walsh-Fourier series	Yu. Shteinikov: On the product and quotient sets of rational numbers and integers

15:40-16:10 Coffee break

16:10	D.M. Israfilov: Approximation problems in the variable exponent spaces	K. Gyarmati: On irregularities of distribution of binary sequences relative to arithmetic progression
16:35	P.S. Patra: Images of some subspaces of $L^2(\mathbb{R}^m)$ under Grushin and Hermite semigroup	A. Torgashova: One-sided mean approximation on the Euclidean sphere to the characteristic function of a spherical layer by algebraic polynomials

August 26, Saturday. Venue: PAB

09:35	A. Babenko: Interrelation between integral and uniform approximations
10:00	A. Kivinukk: On approximation processes defined by the cosine operator function in a Banach space
10:45	F. Móricz: Moment inequalities for the maxima of partial sums in the probability theory with applications in the theory of orthogonal series

11:10-11:40 Coffee break

11:40	Gy. Gát: Convergence of subsequences of partial sums of trigonometric Fourier series
12:25	M. Kolountzakis: Measurable Steinhaus sets do not exist for finite sets or the integers in the plane

13:10-14:30 Lunch break

14:30	B. Nagy: On a potential theoretic minimax problem on the torus	S. Öztop: Some properties of Orlicz algebras on groups
14:55	R. Toledo: The boundedness of the L^1-norm of Walsh-Fejér kernels	C Sivaramakrishnan: On images of Dunkl-Sobolev spaces in $L^2(\mathbb{R}, \|u\|^{2r}e^x dx)$ under Schrödinger semigroup
15:20	B. Király: Construction of orthogonal and biorthogonal product systems	K. Nagy: Some properties of Marcinkiewicz means with respect to Walsh system

15:45-16:15 Coffee break

16:15	A. Testici: Approximation by Poisson polynomials in Smirnov classes with variable exponent	M. Gaál: Integral comparsions of nonnegative, positive definite functions on locally compact groups
16:40	Á. Pilgermajer: Lebesgue functions of rational interpolations of non-band-limited functions	A. Debernardi: Weighted norm inequalities for integral transforms with kernels bounded by power functions

August 27, Sunday. Excursion with wine tasting (optional)

August 28, Monday. Venue: University of Pécs, Building of Faculty of Arts

09:15	S. Tikhonov: Doubling condition at the origin for non-negative positive definite functions
10:00	D. Gorbachev: Nikolskii constants for polynomials on the unit sphere

10:45-11:00 Photo

11:00-11:30 Coffee break

11:40	S. Koumandos: Positive trigonometric integrals associated with some Lommel functions of the first kind
12:15	I. Prause: Arctic curves beyond the arctic circle

13:00-14:30 Lunch break

14:30	T.H. Kaptanoğlu: Inclusions among Bergman-Besov and Bloch-Lipschitz spaces and H^∞ on the ball of $\mathbb{C}^N$
15:15	E. Berestova: The Plancherel-Pólya inequality for entire functions of exponential type in $L^2(\mathbb{R})$

15:40-16:10 Coffee break

16:10	N. Memić: Topics on Nörlund logarithmic means
16:35	D. Virosztek: A short proof of a duality theorem and another application of an intersection formula on dual cones

August 29, Tuesday. Venue: PAB

09:15	R.R. Akopyan: Optimal recovery of derivative of an analytic function
10:00	M. Pap: Slice regular Malmquist-Takenaka system in the quaternionic Hardy spaces
10:25	P. Glazyrina: Szegő-Taikov inequality for conjugate polynomials

10:50-11:30 Coffee break

11:30	L. Székelyhidi: Spectral synthesis on affine groups
12:15	I. Berkes: Trigonometric series with random gaps

13:00-14:30 Lunch break

14:30	I. Simon: Almost everywhere convergence of sequences of (C, α) means of m-adic Fourier series of integrable functions
14:55	M. Deikalova: On Nikol'skii type inequality between the uniform norm and weighted integral q-norm on an interval for polynomials in terms of Bessel functions
15:20	V. Fülöp: A note on the magnitude of Fourier transform and Walsh-Fourier transform

15:45-16:15 Coffee break

16:15	T. Eisner: The Voice transform on the Blaschke group of the $\mathbb{I} = (0, 1)$
16:40	Á. Horváth: The Dirichlet problem in weighted norm

August 30, Wednesday. Venue: PAB

09:15	N.A. Kuklin: Polynomial optimization methods for extremal problems in discrete geometry on Euclidean sphere
10:00	M. Matolcsi: Orthogonal Latin squares in low dimensions
10:45	L. Tóth: Ramanujan-Fourier expansions of arithmetic functions of several variables

11:10-11:40 Coffee break

11:40	N. Hegyvári: Character sum estimations via Additive Combinatorics
12:05	A. Bonami: Spectral decay of finite Fourier transforms and related random matrices

12:50-13:00 Closing

13:00- Lunch and optional programs

Optimal recovery of derivative of an analytic function

Akopyan, Roman R.

Let G be a simply connected bounded domain on the complex plane with boundary Γ which is a closed rectifiable Jordan curve. Denote by γ an arbitrary Lebesgue measurable subset of Γ of positive measure. Consider the Hardy space $H(G)$ of functions analytic and bounded on the domain G with the norm $\|f\|_{H(G)} = \sup\{|f(z)| : z \in G\}$.

Denote by Υ_z^1 the functional which is defined on the subspace of $L^\infty(\gamma)$ formed by functions that are boundary values on γ of functions from the space $H(G)$ and which assigns the value of derivative of a given analytic function at the point z of G to the boundary values of the function on γ, i.e., the functional given by the equation $\Upsilon_z^1 f = f'(z)$. In the space $H(G)$, consider the class Q of functions satisfying the condition $\|f\|_{L^\infty(\Gamma\setminus\gamma)} \leq 1$. We will discuss several related extremal problems for the functional Υ_z^1 on the class Q.

(1) The study of the function of real variable $\delta \in [0, \infty)$ defined by the relation

$$\omega(\delta) = \omega(\delta; \Upsilon_z^1, Q) = \sup\left\{|f'(z)| : f \in Q, \|f\|_{L^\infty(\gamma)} \leq \delta\right\},$$

which is referred to as the modulus of continuity of the functional Υ_z^1 on the class Q. Along with the values of the quantity $\omega(\delta)$, an extremal function at which the upper bound is attained is also of interest.

(2) The problem of optimal recovery of the value of derivative of an analytic function at a point z of the domain. Let, for an unknown function f in the class Q, a function $q \in L^\infty(\gamma)$ be given such that $\|f - q\|_{L^\infty(\gamma)} \leq \delta$. In other words, the boundary values of the function f are given with error δ on the part γ of the boundary. Our aim is to recover the value $f'(z)$ from q in the best possible (optimal) way. For the set $\mathcal{R}$ of methods of recovery from which the optimal one is chosen we take the set $\mathcal{O}$ of all possible functionals or the set $\mathcal{L}$ of all linear functionals or the set $\mathcal{B}$ of all bounded functionals on $L^\infty(\gamma)$. The precise statement of the problem is as follows. For a number $\delta > 0$ and a method of recovery $T \in \mathcal{R}$, define the value of the error of the method by the formula

$$\mathcal{U}(T, \delta) = \sup\left\{|f'(z) - Tq| \,:\, f \in Q,\, q \in L^\infty(\gamma),\, \|f - q\|_{L^\infty(\gamma)} \leq \delta\right\}.$$

Then,

$$\mathcal{E}_{\mathcal{R}}(\delta) = \inf\left\{\mathcal{U}(T, \delta) \,:\, T \in \mathcal{R}\right\}$$

is the quantity of optimal recovery of the value of derivative of an analytic function at the point z (or, equivalently, of optimal recovery of the functional Υ_z^1) by the methods of recovery $\mathcal{R}$ on functions of the class Q from their boundary values γ specified with the error δ. The problem is to find the quantity $\mathcal{E}_{\mathcal{R}}(\delta)$ and to define an optimal way of the recovery, i.e., a functional at which the lower bound is attained. This problem is a special case of the problem of optimal recovery of operators on a class of elements of a Banach space from incomplete or imperfect information.

(3) The problem of the best approximation of a functional Υ_z^1 by bounded linear functionals. The precise statement of the problem is as follows. Let $\mathcal{L}(N)$ be the

set of bounded linear functionals on $L^\infty(\gamma)$ whose norm does not exceed the number $N > 0$. The quantity

$$U(T) = \sup\left\{|f'(z) - Tf| \,:\, f \in Q\right\}$$

is the deviation of a functional $T \in \mathcal{L}(N)$ from the functional Υ_z^1 on the class Q. Correspondingly, the quantity

$$E(N) = \inf\left\{U(T) \,:\, T \in \mathcal{L}(N)\right\}$$

is the best approximation of the functional Υ_z^1 by the set of bounded linear functionals $\mathcal{L}(N)$ on the class Q. The problem is to calculate the quantity $E(N)$ and find an extremal functional at which the lower bound is attained. This problem is a special case of Stechkin's problem on the approximation of an unbounded operator by bounded linear operators on a class of elements of a Banach space [Ste67].

The history of the study of these problems and the results on their relationship see in [Ste67]. The monograph [Osi00] is devoted to problems of optimal recovery on classes of analytic functions.

This work was supported by the Russian Foundation for Basic Research (project no. 15-01-02705) and by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1).

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Interrelation between integral and uniform approximations

Babenko, Aleksandr

We will discuss best approximations to certain individual functions, including trigonometric fractions of a special type, by trigonometric polynomials in the integral and uniform metrics. In particular, we plan to point out the relationship between alternance points (in the problem of least deviation of trigonometric fractions from zero in the uniform metric) with canonical sets of points of given cardinality that play a key role in the theory of integral approximation of functions by trigonometric polynomials.

Joint work with Yu. V. Kryakin.

This work was supported by the Russian Foundation for Basic Research (project no. 15-01-02705) and by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1).

The Plancherel-Pólya inequality for entire functions of exponential type in $L^2(\mathbb{R})$

Berestova, Ekaterina

We investigate the inequality $\sum_{k\in\mathbb{Z}} |f(k)|^p \leq c_p(\nu)\|f\|^p_{L^p(\mathbb{R})}$ on the set of entire functions f of exponential type $\nu > 0$ that belong to the space $L^p(\mathbb{R})$. It is known that $c_2(\nu) = 1$ for $\nu \leq \pi$, this is the classical theorem from harmonic analysis and communication theory proved by Plancherel-Pólya and independently by Whittaker, Kotelnikov and Shannon. The value $c_p(\nu)$ and its generalizations studied by S.N. Nikolskii, R.P. Boas, D.L. Donoho and B.F. Logan, S. Norvidas. Donoho and Logan for $p = 1, 2$ and S. Norvidas for $1 < p < 2$ received the following estimate $c_p(\nu) \leq \dfrac{[\delta]+1}{2\delta\|\cos(\nu\delta\cdot)\|^p_{L^p[0,1/2]}}$, $0 < \nu\delta < \pi$, in particular, $c_2(\nu) \leq \dfrac{2\nu([\delta]+1)}{\nu\delta+\sin(\nu\delta)}$. We proved that $c_2(\nu) = \nu/\pi$, if ν/π is a positive integer.

This work was supported by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1)

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Trigonometric series with random gaps

Berkes, István

In their pioneering work “Some random series of functions I-III” (1931/32), Paley and Zygmund started the study of trigonometric series with random coefficients and by now there exists a wide and nearly complete theory of such series. In contrast, there exists no systematic theory of trigonometric series $\sum(a_k \cos n_k x + b_k \sin n_k x)$ with random frequencies n_k, even though such series provide a powerful tool for constructions and counterexamples in analysis, combinatorics, number theory and probability theory. In this lecture we discuss the asymptotic behavior of some classes of such series, in particular series where the gaps $n_{k+1} - n_k$ are i.i.d. positive random variables i.e. $(n_k)_{k\geq 1}$ is an increasing random walk. If the distribution of the gaps $n_{k+1} - n_k$ is absolutely continuous, then the analogue of Carleson's theorem holds for $\sum c_k f(n_k x)$ for any smooth periodic f with mean zero and the partial sums $\sum_{k=1}^{N} f(n_k x)$ satisfy, with probability one, the central limit theorem and the law of the iterated logarithm. On the other hand, if the $n_{k+1} - n_k$ are integer valued, the previous results break down and the asymptotic properties of the considered partial sums and of the discrepancy of $\{n_k x\}$ depend sensitively on the Diophantine approximation properties of x.

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Goldbach representations with congruences

Bhowmik, Gautami

The classical Goldbach problem examines the possibility of expressing even integers as the sum of two primes. The associated generating function gives satisfactory asymptotics under the Riemann Hypothesis and obtaining sufficiently good error terms unconditionally is expected to solve the famous hypothesis. In joint work with K. Halupczok, K. Matsumoto and Y. Suzuki, we consider the case where the summands are in arithmetic progression and obtain asymptotics assuming now a conjecture on distinct zeros of Dirichlet L-functions. The existence of good error terms gives information on the the location of zero free regions of these functions and possible Siegel zeros.

Approximation by Θ-means of Walsh-Fourier series

Blahota, István

In this lecture we discuss the behaviour of Θ-means of (one- and two-dimensional) Walsh series of a function in L^p $(1 \leq p \leq \infty)$.

We investigate the rate of the approximation by this means, in particular, in $\mathrm{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \leq p \leq \infty$. In case $p = \infty$ by L^p we mean C, the collection of the continuous functions.

Our main theorems give a common generalization (and two-dimensional analogues) of two results of Móricz, Siddiqi on Nörlund means [MS92] and Móricz, Rhoades on weighted means [MR96].

Our main two-dimensional theorem generalizes two result of Nagy on Nörlund means and weighted means of the cubical partial sums of double Walsh-Fourier series [Nag10a, Nag10b].

Joint work with Károly Nagy.

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Spectral decay of finite Fourier transforms and related random matrices

Bonami, Aline

The finite Fourier transform operator, and in particular its singular values, have been extensively studied in relation with band-limited functions. Recall that, for m a positive number, the finite Fourier transform $\mathcal{F}_m$ is defined on $L^2(-1/2, +1/2)$ by

$$\mathcal{F}_m(f)(y) = \sqrt{m} \int_{-1/2}^{+1/2} \exp(2i\pi myz) f(z) dz, \qquad |y| < 1/2.$$

We study singular values of an $n \times n$ matrix, which may be seen as a random discretization of $\mathcal{F}_m$. This type of matrices has been introduced by Desgroseilliers, Lévêque and Preissmann as an approximate model of a wireless communication network. We prove that, with high probability, the sequence of singular values of the random matrix is close to the sequence of singular values of the finite Fourier transform itself. As an application, we prove that the number of degrees of freedom is well approximated by m.

Our main tool is a study of kernel matrices $\kappa(Y_j, Y_k)$, where Y_j is a sample of some probability law and κ is a positive semi-definite kernel. Some results on kernel matrices are of independent interest.

This talk will also be the occasion to revisit the decay of the spectrum of sinc operators, which are given on $L^2(-1/2, +1/2)$ by the kernel

$$\operatorname{sinc}(m(x-y)) = \frac{\sin m\pi(x-y)}{\pi(x-y)}.$$

It is well-known that the spectrum of this operator is first larger than 1/2 up to m, then decreases super-exponentially. We will give some precise estimates and show how this is related to Remez type inequalities on trigonometrical polynomials.

Results on random matrices are issued from a joint work with Abderrazek Karoui. The decay of the spectrum of the sinc operator is the object of work in progress with Philippe Jaming and Abderrazek Karoui.

Weighted norm inequalities for integral transforms with kernels bounded by power functions

Debernardi, Alberto

For $1 < p \le q < \infty$ and $\beta, \gamma \in \mathbb{R}$, we study necessary and sufficient conditions for the weighted norm inequality

$$\|y^{-\beta} Ff\|_{L^q(\mathbb{R}_+)} \le C\|x^\gamma f\|_{L^p(\mathbb{R}_+)} \tag{1}$$

to hold with absolute C, where

$$Ff(y) = y^c \int_0^\infty x^b f(x) K(x,y)\, dx, \qquad b, c \in \mathbb{R},$$

and K is a continuous kernel satisfying the estimate

$$|K(x,y)| \lesssim \min\{(xy)^{b_1}, (xy)^{b_2}\}, \tag{2}$$

with $b_1, b_2 \in \mathbb{R}$ (a priori). Examples of F are the Fourier, cosine and sine transforms, the classical Hankel transform, and the $\mathscr{H}_\alpha$ transform, defined as

$$\mathscr{H}_\alpha f(y) = \int_0^\infty (xy)^{1/2} f(x) \mathbf{H}_\alpha(xy)\, dx, \qquad \alpha > -1/2,$$

where $\mathbf{H}_\alpha$ is the Struve function of order α. Despite our approach is simple as it only consists on applying Hardy's inequality together with estimate (2), we readily find one limitation; we have to assume $b_1 > b_2$ in (2), thus the Fourier and cosine transforms are already out of our scope. Also, for the other mentioned transforms, the sufficient conditions we can derive for (1) are not optimal (see [DC08], [GLT], [Muc83] and [Roo80]), although they are sharp for transforms with kernels satisfying $K(x,y) \asymp \min\left\{(xy)^{b_1}, (xy)^{b_2}\right\}$.

Nevertheless, we have applications of our main result. In the first place, we consider kernels of the type

$$K(x,y) = \sum_{k=0}^{\infty} a_k (xy)^{mk+d}, \qquad \{a_k\} \subset \mathbb{C}, \quad m \in \mathbb{N},\, d \geq 0,$$

and obtain more relaxed sufficient conditions for (1) to hold provided that enough moments of f vanish, along the lines of [SW87]. Secondly, we apply our approach restricting f to satisfy general monotonicity assumptions. In this case we also get less restrictive sufficient conditions for (1) to hold, and those are sharp when F is one of the aforementioned transforms (cf. [DCGT13, GLT11] for the Hankel and Fourier transforms).

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On Nikol'skii type inequality between the uniform norm and weighted integral q-norm on an interval for polynomials in terms of Bessel functions

Deikalova, Marina

Let J_ν be the Bessel functions (of the first kind) of order $\nu > -1$. The function $j_\nu(z) = \Gamma(\nu+1)\,(2/z)^\nu\, J_\nu(z)$ is called the modified Bessel function; this is an entire function. Let $\{\lambda_m = \lambda_m^{(\nu)}\}_{m=1}^\infty$ be positive zeros of the Bessel function arranged in ascending order of magnitude (see [Wat95, Ch. XV], [EMOT53, Ch. 7, Sect. 7.9], [Vla71, Sect. 23]). Series in terms of the system of functions $\{j_\nu(\lambda_m x)\}_{m=1}^\infty$ are of great importance in mathematical physics (see, in particular, [Wat95, Ch. XVIII], [Vla71, Sect. 23]).

For $n \geq 1$, denote by $\mathcal{P}_n$ the set of functions of the form

$$p_n(x) = \sum_{k=1}^{n} a_k j_\nu(\lambda_k x).$$

The aim of the talk is to discuss the sharp Nikol'skii type inequality

$$\|p_n\|_{C[0,1]} \leq M_n \|p_n\|_{L_q([0,1],x^{2\nu+1})}, \quad p_n \in \mathcal{P}_n,$$

on the interval $[0,1]$ between the uniform norm and the norm

$$\|f\|_{L_q([0,1],x^{2\nu+1})} = \left(\int_0^1 |f(x)|^q x^{2\nu+1} dx\right)^{1/q}$$

of the space $L_q = L_q([0,1], x^{2\nu+1})$ on the set $\mathcal{P}_n$ for $\nu > -1/2$.

The talk is based on joint research with V. Arestov, A. Babenko, and Á. Horváth.

This work was supported by the Russian Foundation for Basic Research (project no. 15-01-02705), by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1), and by the Russian Academic Excellence Project (agreement no. 02.A03.21.0006 of August 27, 2013, between the Ministry of Education and Science of the Russian Federation and Ural Federal University).

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The Voice transform on the Blaschke group of the $\mathbb{I} = (0, 1)$

Eisner, Tímea

The Blaschke-functions play an important role in system identification. These functions form a group with respect to the composition of functions. In 2006 and 2007 the Voice transform on the Blaschke group was introduced by M. Pap and F. Schipp. ([PS06]; [PS08]; [Sch14])

Instead of the Blaschke group $(\mathbb{B}, \circ)$, we consider the analogue of it on the interval $\mathbb{I} := (-1, 1)$. We prove that the representation which induces this special voice-transform, is unitary.

Using this representations and taking the images of Legendre polynomials through the representation we obtain a new rational orthonormal system on the Hilbert Space $L^2(\mathbb{I})$.

We will construct the rational analogue of the Jacobi and Chebyshev systems taking their image through the representation, obtaining in this way other new orthogonal systems.

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On the abundance of Fourier standard spaces

Feichtinger, Hans G.

The usual description of the Fourier transform (over)emphasizes the role of L^p-spaces. L^1 is used to define the Fourier integral and convolution, and L^2 is needed to express the fact that the Fourier transform is a unitary mapping (over the torus, but also for the Euclidean space R^d). But the drawback it the asymmetry, which becomes apparent when proving the Fourier inversion theorem.

Time-frequency Analysis resp. *Gabor Analysis* suggest to take a more symmetric view on the time resp. frequency side of a function by taking a *time-frequency perspective*. In this context also the so-called Segal algbra S_0 and its dual play a significant role, e.g. in order to derive robustness results for Gabor expansions.

With S_0 being the smallest element in this family of isometrically time-frequency shift-invariant spaces (and correspondingly its dual space S_0' being the largest one of its kind) it appears to be natural to study the whole family of such spaces, which we call *Fourier standard spaces.*

We will exploit the richness of this family of spaces, which includes aside of L^p-spaces also the Wiener amalgam spaces $W(L^p, \ell^q)$, the modulation spaces $M^{p,q}$, the space of convolution kernels acting boundedly on L^p and so on. Nevertheless one can derive quite general results for the spaces in this family and show that this new perspective is quite useful, also from the point of view of classical analysis. For example one can characterize the reflexive spaces in this family or the ones which are dual spaces.

A note on the magnitude of Fourier transform and Walsh-fourier transform

Fülöp, Vanda

It appears that the study of the order of magnitude of the trigonometric Fourier-coefficients for the functions of bounded variation and of various classes of generalized bounded variations ($BV^{(p)}$, ϕBV, ΛBV, $\Lambda BV^{(p)}$, $\phi\Lambda BV$, $p \geq 1$) has been carried out as well as the study of the Walsh-Fourier coefficients. But such a study for the Fourier transform and for the Walsh-Fourier transform has not yet been done. In this note we give an estimate for the order of magnitude of the Fourier transform and for the Walsh-Fourier transform for functions of BV on $\mathbb{R}$ or $\mathbb{R}^+$, respectively. After that we generalize these results for functions of BV_V on $\mathbb{R}^N$ or $(\mathbb{R}^+)^N$, respectively ($N \in \mathbb{N}$).

Joint work with B.L. Ghodadra.

Integral comparsions of nonnegative, positive definite functions on locally compact groups

Gaál, Marcell

In this work we mainly discuss the following general question. Let μ, ν be two regular Borel measures of finite total variation. Under what conditions do we have a constant C (and how the admissible constants C can be characterized, and what is the best value, if any) satisfying that $\int f d\nu \leq C \int f d\mu$ whenever f is a continuous nonnegative positive definite function vanishing at infinity? The question (on $\mathbb{R}$) is due to G. Halász.

We discuss the problem first in general locally compact Abelian groups, and then apply the general results to the most interesting special case when μ, ν are the traces of the usual Lebesgue measure over centered and arbitrary intervals, respectively. This special case was earlier investigated by Shapiro, Montgomery, Halász and Logan, and is a close companion of the more familiar problem of Wiener.

Joint work with Szilárd Gy. Révész.

Convergence of subsequences of partial sums of trigonometric Fourier series

Gát, György

Research supported by the Hungarian National Foundation for Scientific Research (OTKA), grant no. K111651.

In the theory of trigonometric Fourier series it is of great interest that how to reconstruct the function from the partial sums of its Fourier series. In 1966 Carleson [Car66] showed that if $f \in L^2$, then the partial sums converge to the function almost everywhere. It is also a fundamental question that how to reconstruct a function belonging to L^1 from the partial sums of its Fourier series. Lebesgue showed that for each integrable function we have the almost everywhere convergence of Fejér means $\sigma_n f = \frac{1}{n+1}\sum_{m=0}^{n} S_m f \to f$.

It is also of prior interest that what can be said - with respect to this reconstruction issue - if we have only a subsequence of the partial sums. In 1982 Totik showed [Tot82] that for each subsequence (n_j) of the sequence of natural numbers there exists an integrable function f such that $\sup_j |S_{n_j} f| = +\infty$ everywhere.

In 1936 Zygmunt Zalcwasser [Zal36] asked how “rare” can a sequence of integers (n_j) be such that

$$\frac{1}{N}\sum_{j=1}^{N} S_{n_j} f \to f$$

a.e. for every function $f \in L^1$. In this talk we give an answer for this question.

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Szegő-Taikov inequality for conjugate polynomials

Glazyrina, Polina

We consider a linear combination of a trigonometric polynomial and its conjugate. A sharp estimate for the uniform norm of the linear combination via the uniform norm of the polynomials is obtained.

This work was supported by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1)

Nikolskii constants for polynomials on the unit sphere

Gorbachev, Dmitry

We first study the asymptotic behavior of sharp Nikolskii constant

$$C(n,d,p,q) := \sup_{f\in\Pi_n^d,\, \|f\|_{L^p(\mathbb{S}^d)}=1} \|f\|_{L^q(\mathbb{S}^d)}$$

for $0<p<q\le\infty$ as $n\to\infty$, where Π_n^d denotes the space of all spherical polynomials f of degree at most n on the unit sphere $\mathbb{S}^d\subset\mathbb{R}^{d+1}$.

1. *We prove that for* $0<p<\infty$ *and* $q=\infty$,

$$\lim_{n\to\infty}\frac{C(n,d,p,\infty)}{n^{d/p}}=\mathcal{L}(d,p,\infty),$$

and for $0<p<q<\infty$,

$$\liminf_{n\to\infty}\frac{C(n,d,p,q)}{n^{d\,(1/p-1/q)}}\ge\mathcal{L}(d,p,q),$$

where the constant $\mathcal{L}(d,p,q)$ *is defined for* $0<p<q\le\infty$ *by*

$$\mathcal{L}(d,p,q) := \sup_{f\in\mathcal{E}_p^d,\, \|f\|_{L^p(\mathbb{R}^d)}=1} \|f\|_{L^q(\mathbb{R}^d)},$$

with $\mathcal{E}_p^d$ *denoting the set of all entire functions* $f\in L^p(\mathbb{R}^d)$ *of spherical exponential type at most* 1. *These results extend the recent results of Levin and Lubinsky for trigonometric polynomials on the unit circle.*

Next, we estimate the normalized Nikolskii constant

$$L_d := \frac{|\mathbb{S}^d|\Gamma(d+1)}{2}\,\mathcal{L}(d,1,\infty).$$

It was known that $C(n,d,p,q)\le(|\mathbb{S}^d|^{-1}\dim\Pi_n^d)^{1/p-1/q}$, $0<p\le2$. This implies $L_d\le1$. The lower estimate of $C(n,d,1,\infty)$ obtained by Deikalova gives $e^{-d}\le L_d$.

2. *We prove the following lower and upper estimates:*

$$2^{-d}\le L_d\le {}_1F_2\Big(\frac{d}{2};\frac{d}{2}+1,\frac{d}{2}+1;-\frac{\beta_d^2}{4}\Big),$$

where ${}_1F_2$ *and* β_d *denote the hypergeometric function, and the smallest positive zero of the Bessel function* $J_{d/2}$*, respectively. In particular, this implies that the constant* L_d *decays exponentially fast as* $d\to\infty$*:*

$$2^{-d}\le L_d\le(\sqrt{2/e})^{d\left(1+O(d^{-2/3})\right)},$$

where $\sqrt{2/e}=0.85776\cdots$.

3. *Furthermore, we find the following sharp constant in the Nikolskii inequality for nonnegative functions*

$$\sup_{f\in\mathcal{E}_1^d,\, f\ge0}\frac{\|f\|_{L^\infty(\mathbb{R}^d)}}{\|f\|_{L^1(\mathbb{R}^d)}}=\frac{1}{2^{d-1}|\mathbb{S}^d|\Gamma(d+1)}$$

as well as in the Bernstein–Nikolskii inequality

$$\sup_{f\in\mathcal{E}_1^d,\, f\geq 0} \frac{\|\Delta f\|_{L^\infty(\mathbb{R}^d)}}{\|f\|_{L^1(\mathbb{R}^d)}} = \frac{1}{2^d|\mathbb{S}^d|\Gamma(d+1)(d+2)}.$$

4. Finally, we observe that for $d \geq 2$, the asymptotic order of the usual Nikolskii inequality on $\mathbb{S}^d$ can be significantly improved in many cases, for lacunary spherical polynomials of the form $f = \sum_{j=0}^m f_{n_j}$ with f_{n_j} being a spherical harmonic of degree n_j and $n_{j+1} - n_j \geq 3$. As is well known, for $d = 1$, the Nikolskii inequality for trigonometric polynomials on the unit circle does not have such a phenomenon.

Joint work with Feng Dai and Sergey Tikhonov.

F. D. was supported by NSERC Canada under the grant RGPIN 04702 Dai. D. G. was supported by the RFBR (no. 16-01-00308). S. T. was partially supported by MTM 2014-59174-P and 2014 SGR 289.

On irregularities of distribution of binary sequences relative to arithmetic progression

Gyarmati, Katalin

In 1964 K. F. Roth initiated the study of irregularities of distribution of binary sequences relative to arithmetic progressions and since that numerous papers have been written on this subject. In the applications one needs binary sequences which are well distributed relative to arithmetic progressions, in particular, in cryptography one needs binary sequences whose short subsequences are also well-distributed relative to arithmetic progressions. Thus with my coauthors Cecile Dartyge and András Sárközy we introduced weighted measures of pseudorandomness of binary sequences to study this property. We studied the typical and minimal values of this measure for binary sequences of a given length. We also gave constructive bounds for the minimal values.

Character sum estimations via Additive Combinatorics

Hegyvári, Norbert

In the last decade character sum estimations focused the interest of many researchers in the domain of additive combinatorics. These estimations are important in computer sciences, coding theory e.t.c.

In the present talk we show how could help additive combinatorics in this area: we concentrate to multilinear exponential sums in additive sets; character sums in Hilbert cubes, and the problems of covering polynomials in prime fields.

Matrices with multiplicative entries

Hilberdink, Titus

We study operators which have (infinite) matrix representation whose entries are multiplicative functions of two variables. We show that such operators are infinite tensor products over the primes. Applications to finding the eigenvalues explicitly of arithmetical matrices are given; also multiplicative Toeplitz and Hankel operators are discussed.

The Dirichlet problem in weighted norm

Horváth, Ágota P.

We study the classical Dirichlet problem in the disc with the weighted uniform norm for the weight function $w(x) = v(x) \prod_{j=1}^{s} \left|\sin\left(\frac{x-x_j}{2}\right)\right|^{\lambda_j}$, $\{\lambda_j\}_{j=1}^{s}$ are positive numbers and v is a strictly positive continuous function on the circle. Remarkably the problem has solution if and only if none of the numbers $\{\lambda_j\}_{j=1}^{s}$ is natural.

Joint work with K. S. Kazarian.

Approximation problems in the variable exponent spaces

Israfilov, Daniyal M.

In this talk we discuss the approximation problems in the variable exponent Lebesgue spaces, which are the generalization of the classical Lebesgue spaces. In particular, the following problems are considered:

- The direct and inverse problems of approximation theory,
- The constructive characterizations problems,
- Simultaneous approximation problems,
- Maximal convergence problems in the variable exponent Smirnov classes.

The weighted versions of these problems also are investigated. Moreover, the Smirnov class of analytic functions given on the domains of complex plane is defined and under some restrictive conditions upon to the exponents the solvability of the above mentioned problems in these classes are discussed.

Joint work with Ahmet Testici.

This work was supported by TUBITAK grant 114F422: "*Approximation Problems in the Variable Exponent Lebesgue Spaces*".

Some extremal problems for Fourier transform on hyperboloid

Ivanov, Valerii

Let $d \in \mathbb{N}$, $d \geq 2$, and suppose that $\mathbb{R}^d$ is d-dimensional real Euclidean space with inner product $(x, y) = x_1 y_1 + \ldots + x_d y_d$, and norm $|x| = \sqrt{(x, x)}$, $\mathbb{S}^{d-1} = \{x \in \mathbb{R}^d : \ |x| = 1\}$ is the Euclidean sphere, $\mathbb{R}^{d,1}$ is $(d + 1)$-dimensional real pseudoeuclidean space with bilinear form $[x, y] = -x_1 y_1 - \ldots - x_d y_d + x_{d+1} y_{d+1}$, $\mathbb{H}^d = \{x \in \mathbb{R}^{d,1} : [x, x] = 1, x_{d+1} > 0\}$ is the upper sheet of two sheets hyperboloid, $d(x, y) = \operatorname{arc\,cosh}[x, y]$ is the distance between $x, y \in \mathbb{H}^d$, $o = (0, \ldots , 0, 1) \in \mathbb{H}^d$, $r > 0$, $B_\tau = \{x \in \mathbb{H}^d : d(x, o) \leq r\}$. The pair $\big(\mathbb{H}^d, d(\cdot, \cdot)\big)$ is known as the Lobachevskii space.

Let $t, \lambda \in \mathbb{R}_+$, $\eta, \xi \in \mathbb{S}^{d-1}$, $x = (\sinh t\, \eta, \cosh t) \in \mathbb{H}^d$, $y = (\lambda, \xi) \in \mathbb{R}_+ \times \mathbb{S}^{d-1}$. Let $d\mu(t) = 2^{d-1} \sinh^{d-1} t\, dt$, $d\sigma(\lambda) = 2^{3-2d}\Gamma^{-2}(d/2)|\Gamma((d - 1)/2 + i\lambda|^2|\Gamma(i\lambda)|^{-2}d\lambda$ be the Lebesgue measures on $\mathbb{R}_+$, let $d\omega(\eta) = |\mathbb{S}^{d-1}|^{-1}\, d\eta$ be the probability measure on $\mathbb{S}^{d-1}$ and let $d\nu(x) = d\mu(t)d\omega(\eta)$, $d\tau(y) = d\sigma(\lambda)d\omega(\xi)$ be the measures on $\mathbb{H}^d$ and $\Omega^d = \mathbb{R}_+ \times \mathbb{S}^{d-1}$, respectively. Let $Qg(\lambda) = \int_{\mathbb{S}^{d-1}} g(y)\, d\omega(\xi)$, $y = (\lambda, \xi) \in \Omega^d$, be the averaging operator.

The direct and inverse Fourier transforms in Lobachevskii space have the forms $\mathcal{F}f(y) = \int_{\mathbb{H}^d} f(x)[x, \xi']^{-\frac{d-1}{2}-i\lambda}\, d\nu(x)$, $\quad \mathcal{F}^{-1}g(x) = \int_{\Omega^d} g(y)[x, \xi']^{-\frac{d-1}{2}+i\lambda}\, d\tau(y)$, where $\xi' = (\xi, 1)$, $\xi \in \mathbb{S}^{d-1}$.

We solve the following extremal problems for Fourier transforms on $\mathbb{H}^d$ and Ω^d.

The Turán problem. Calculate the quantity $T(r, \mathbb{H}^d) = \sup Q(\mathcal{F}f)(0)$, if $f \in C_b(\mathbb{H}^d)$, $f(o) = 1$, $\operatorname{supp} f \subset B_r$, $\mathcal{F}f(y) \geq 0$.

The Fejér problem. Calculate the quantity $F(r, \mathbb{R}_+) = \sup Qg(0)$, if $g = \mathcal{F}f \in L_1(\Omega^d, d\tau)$, $g(y) \geq 0$, $f \in C_b(\mathbb{H}^d)$, $f(o) = 1$, $\operatorname{supp} f \subset B_r$.

The Delsarte problem. Calculate the quantity $D(r, s, \mathbb{H}^d) = \sup \mathcal{F}^{-1}g(o) = \sup f(o)$, if $g = \mathcal{F}f \in L_1(\Omega^d, d\tau)$, $Qg(0) = 1$, $g(\lambda, \xi) \leq 0$, $\lambda \geq s$, $f \in C_b(\mathbb{H}^d)$, $\operatorname{supp} f \subset B_r$, $f(x) \geq 0$.

The Boman problem. Let $y = (\lambda, \xi) \in \Omega^d$. Calculate the quantity $B(r, \mathbb{H}^d) = \inf \int_{\Omega^d} (\lambda^2 + ((d - 1)/2)^2)g(y)\, d\tau(y)$, if $g = \mathcal{F}f \in L_1(\Omega^d, d\tau)$, $g(y) \geq 0$, $f \in C_b(\mathbb{H}^d)$, $f(o) = 1$, $\operatorname{supp} f \subset B_r$.

Let g be a real function on Ω^d and $\Lambda(g) = \sup\{\lambda > 0 : \ g(\lambda, \xi) > 0, \ \xi \in \mathbb{S}^{d-1}\}$.

The Logan problem. Calculate the quantity $L(r, \mathbb{H}^d) = \inf \Lambda(g)$, if $g = \mathcal{F}f \in L_1(\Omega^d, d\tau)$, $g(y) \not\equiv 0$, $f \in C_b(\mathbb{H}^d)$, $\operatorname{supp} f \subset B_r$, $f(x) \geq 0$.

By averaging functions over the sphere we reduce these problems to the similar problems for the Jacobi transforms on the half-line. The extremal functions are spherical and unique. To solve the one-dimensional problems we apply the Gauss and Markov quadrature formulae for even entire functions of exponential type with nodes at zeros of the Jacobi function [GI15]. The Delsarte problem is solved only under an additional relation between the parameters r, s.

Joint work with Dmitry Gorbachev and Oleg Smirnov.

The authors were supported by RFBR (project 16-01-00308).

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Inclusions among Bergman-Besov and Bloch-Lipschitz spaces and H^∞ on the ball of $\mathbb{C}^N$

Kaptanoğlu, Turgay H.

The Bergman-Besov spaces B^p_q with $q \in \mathbb{R}$ and $p > 0$ consist of holomorphic functions on the unit ball $\mathbb{B}$ of $\mathbb{C}^N$ whose sufficently high-order radial derivatives lie in the Bergman spaces on $\mathbb{B}$ with standard weights $(1-|z|^2)^q$ for which $q > -1$. The Bloch-Lipschitz spaces $\mathcal{B}^\infty_\alpha$ with $\alpha \in \mathbb{R}$ are the $p = \infty$ versions of the B^p_q.

We describe exactly and fully which of the spaces in the title are included in which others. We construct explicit functions in each space that make sure that each containment is strict and best possible. Our constructions involve lacunary series of Ryll-Wojtaszczyk polynomials and atomic decompositions of the spaces among others. Many of the inclusions turn out to be sharper than the Sobolev imbeddings.

This is joint work with A. Ersin Üreyen of Anadolu University, Eskişehir.

Construction of orthogonal and biorthogonal product systems

Király, Balázs

As the product system of conditionally orthogonal systems we will get orthonormed systems or biorthogonal system-pairs. These systems have several useful properties. With these systems we can define efficient interpolation algorithms and the Fourier coefficients with respect to these systems and the partial sums can be computed with an FFT-like fast algorithm. Several well-known systems can be derived this way.

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On approximation processes defined by the cosine operator function in a Banach space

Kivinukk, Andi

Co-authored with Anna Saksa.

Let X be a Banach space, such that for each $f \in X$ one may associate its Fourier partial sums operator

$$S_n f = \sum_{k=0}^{n} P_k f.$$

In this presentation we define in abstract setting certain approximation operators and find the order of approximation via a modulus of continuity defined by a cosine operator function.

Definition 1. *A cosine operator function $T_h : X \to X$ $(h \geq 0)$ is defined by the properties:*
1. $T_0 = I$ (identity operator),
2. $T_{h_1} \cdot T_{h_2} = \frac{1}{2}(T_{h_1+h_2} + T_{|h_1-h_2|})$,
3. $\|T_h f\| \leq T\|f\|$, $0 < T-$ not depending on $h > 0$.

Definition 2. *Let the Rogosinski-type operators $R_{n,h,a} : X \to X$ be defined via the Fourier partial sums and the cosine operator function by*

$$R_{n,h,a} f := aT_h(S_n f) + (1-a)\, T_{3h}(S_n f) \;\; (h \geq 0, a \in R).$$

Remark. *The case $a = 1$ leads to the original Rogosinski operator $R_{n,h}$, which in trigonometric approximation was introduced by W.W. Rogosinski [Rog26].*

Following order of approximation, using the abstract modulus of continuity

$$\omega_k(f, \delta) := \sup_{0 \leq h \leq \delta} \|(T_h - I)^k f\|, \; k \in N,$$

is typical in that framework.

Theorem. *For every $f \in X$, $a \in R$ for the operators $R_{n,h,a} : X \to X$ one has*

$$\|R_{n,h,a} f - f\| \leq (\|R_{n,h,a}\|_{X \to X} + |a|T + |1-a|T)\, E_n(f) + |a|\omega_1(f, h) + |1-a|\omega_1(f, 3h),$$

where $E_n(f)$ is the best approximation of $f \in X$.

The operators, defined via the cosine operator function, are interesting, because they are applicable in approximation by Fourier expansions of different orthogonal systems, in summation of Fourier transforms and in approximation by generalized Shannon sampling operators. Moreover, in some particular cases, we are able to calculate precise values of their operator norms [Kiv17].

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Approximation by multivariate Kantorovich–Kotelnikov sampling operators

Kolomoitsev, Yurii

We study approximation properties of the multivariate Kantorovich–Kotelnikov sampling operator given by

$$\mathcal{K}_j(f;x) = \sum_{k\in\mathbb{Z}^d} \left(|\det M|^j \int_{\mathbb{R}^d} f(u)\widetilde{\varphi}(M^j u + k)du \right) \varphi(M^j x + k),$$

where M is a dilation matrix and $\widetilde{\varphi}$ and φ are appropriate functions. In particular, we consider a wide class of band-limited functions φ including non-integrable ones and a large class of functions $\widetilde{\varphi}$ including both compactly supported and band-limited functions.

Let $\omega_n(f,h)_p$ be the classical modulus of smoothness of order n in $L_p(\mathbb{R}^d)$. Under certain smoothness conditions on φ and $\widetilde{\varphi}$, we prove that the estimate

$$\|f - \mathcal{K}_j(f)\|_{L_p(\mathbb{R}^d)} \le C\omega_n(f, \|M^{-j}\|)_p, \quad f \in L_p(\mathbb{R}^d), \quad j \in \mathbb{Z},$$

holds provided that $D^\beta(1 - \widehat{\varphi}\overline{\widehat{\widetilde{\varphi}}})(\mathbf{0}) = 0$ for all $\beta \in \mathbb{Z}^d_+$, $\|\beta\|_{\ell_1} < n$.

Several examples of the Kantorovich–Kotelnikov operators generated by the sinc-function and its linear combinations are given.

This is a joint work with Maria Skopina (St. Petersburg State University, Russia).

Measurable Steinhaus sets do not exist for finite sets or the integers in the plane

Kolountzakis, Mihalis

A Steinhaus set S for a set B in Euclidean space is a set such that S has exactly one point in common with $t(B)$, for every rigid motion t of Euclidean space. We show here that if B is a finite set of at least two points then there is no such set S which is Lebesgue measurable. An old result of Komjath says that there exists a Steinhaus set for B being the set of integers on the x-axis in 2-space. We also show here that such a set cannot be Lebesgue measurable. We prove the latter result by way of showing that there is no measurable set in the plane which intersects almost every line L at measure 1 (this is still not possible if we ask that the intersection with almost every line is between two positive constants).

Joint work with: Michael Papadimitrakis (Univ. of Crete, Greece)

Positive trigonometric integrals associated with some Lommel functions of the first kind

Koumandos, Stamatis

We prove that

$$\int_0^x t^{\mu-\frac{1}{2}} \log t \, \sin(x-t) \, \mathrm{d}t = \sqrt{x} \, \frac{\mathrm{d}}{\mathrm{d}\mu} \, s_{\mu,\frac{1}{2}}(x) > 0,$$

for all $\mu \geq 1/2$ and $x \geq \pi$, where $s_{\mu,\frac{1}{2}}(x)$ is the Lommel function of the first kind. As an application, we derive positive functional bounds for the functions $s_{\mu,\frac{1}{2}}(x)$, $\mu \geq 1/2$ and $x \geq \pi$. Furthermore, we give estimates for the location of the zeros of the functions $s_{\mu,1/2}(x)$, $x > 0$ for various values of the parameter μ. These results are motivated by the study of the positivity of certain trigonometric sums that play an important role in geometric function theory. We also discuss the log-concavity of the function

$$F_\lambda(x) := \int_0^x (x-t)^\lambda \, \sin t \, dt$$

on $(0, \infty)$.

Polynomial optimization methods for extremal problems in discrete geometry on Euclidean sphere

Kuklin, Nikolai A.

The following extremal geometric problems on Euclidean sphere will be discussed in the talk:

(1) the Thomson problem that asks for minimum electrostatic potential energy configuration of $n \geq 2$ electrons on Euclidean sphere $\mathbb{S}^2$ in three-dimensional Euclidean space $\mathbb{R}^3$;

(2) the problem of packing maximal number $\tau(m, s)$ of equal circles (spherical caps) on the unit sphere $\mathbb{S}^{m-1}$ of m-dimensional Euclidean space $\mathbb{R}^m$ given maximal inner product $s \in [-1, 1)$ between their centers; the well-known kissing number problem is equivalent to the packing problem for $s = 1/2$ — it asks for maximal number $\tau(m) = \tau(m, 1/2)$ of spheres with radius 1 that can simultaneously touch another one without pairwise overlappings in $\mathbb{R}^m$.

Let $(*)$ denote any of these problems.

Currently, the greatest success in $(*)$ has been achieved with the help of Delsarte's scheme and its modifications — the methods that use the notion of positive-definite functions on sphere. Namely, in 1979, Levenshtein and Odlyzko–Sloane independently found $\tau(8) = 240$ and $\tau(24) = 196560$. For $m = 4$, Arestov and Babenko

(1997) showed that Delsarte's scheme gets only the upper bound $\tau(4) \leq 25$; Musin (2003) found $\tau(4) = 24$ by using a modification of Delsarte's scheme. These cases (besides $\tau(2) = 6$ and $\tau(3) = 12$) are the only ones known at the moment.

In 1993, Yudin evolved Delsarte's scheme to get lower bounds for the value of the Thomson problem. He found optimal configurations for $n = 4$ and 6; then, Andreev found an optimal configuration for $n = 12$. In 2010, Schwartz proposed a mathematically rigorous computer-aided solution for $n = 5$. Again, besides the simple cases $n = 2$ and $n = 3$, the Thomson problem is open for other values of n. Unfortunately, we have to accept the fact that Delsarte's scheme has exhausted itself for problems on sphere.

Denote by $\langle q_1, q_2 \rangle$ the standard inner product in $\mathbb{R}^m$; let $\mathbb{S}^{m-1} = \{q \in \mathbb{R}^m \mid \langle q, q\rangle - 1 = 0\}$ be the unit sphere. One more problem for a fixed univariate polynomial h on $\mathbb{S}^{m-1}$ will also be considered in the talk:

$$W_h(q_1, q_2, \dots, q_n) = \sum_{1 \leq i < j \leq n} h(\langle q_i, q_j \rangle);$$

$$\omega_h(n) = \min \bigl\{ W_h(q_1, q_2, \dots, q_n) \mid q_1, q_2, \dots, q_n \in \mathbb{S}^{m-1} \bigr\}. \qquad (**)$$

It turns out that $(**)$ is equivalent to $(*)$ for a specially selected polynomial h, e.g., the following theorem holds.

Theorem. For $m \geq 2$, $n \geq 3$, and $s \in [-1, 1)$, the following conditions are equivalent:

(i) $\tau(m, s) < n$;

(ii) there exists a univariate polynomial h that satisfies the conditions

  (1) $h(t) \leq 0$, $t \in [-1, s]$;

  (2) $\omega_h(n) > 0$.

As $(*)$ and $(**)$ are equivalent (for specific univariate polynomials h), they have the same complexity, but fortunately $(**)$ can be approximated by an infinite series of semidefinite programs (SDP):

$$\omega_h(n, d) = \max \bigl\{ \gamma \in \mathbb{R} \mid W_h - \gamma \in \Sigma_{\leq 2d}\bigl((\mathbb{S}^{m-1})^n\bigr) \bigr\}, \qquad (***)$$

where $f \in \Sigma_{\leq 2d}\bigl((\mathbb{S}^{m-1})^n\bigr)$ means that a multivariate polynomial f can be represented in the form of sum of squares of polynomials of degree $\leq d$ modulo vanishing ideal $\mathcal{I}$ of algebraic variety $(\mathbb{S}^{m-1})^n = \mathbb{S}^{m-1} \times \dots \times \mathbb{S}^{m-1}$. Schmüdgen Positivstellensatz (1991) gives $\omega_h(n, d) \nearrow \omega_h(n)$, $d \to \infty$.

Unfortunately, problem $(***)$ cannot be solved even for 5 points on the sphere $\mathbb{S}^2$ and $\deg(h) = 4$ (this is the Thomson problem for $n = 5$) due to its complexity. I propose an extension of representation theoretic methods that allows us to utilize the symmetry of initial problems $(*)$: they are invariant under

(a) permutations of points;

(b) the same orthogonal transformation that acts on all points;

(c) a composition of (a) and (b).

Actions (a)–(c) constitute the symmetry compact Lie group G of $(\mathbb{S}^{m-1})^n$, which is isomorphic to the direct product of the symmetric group and the orthogonal group: $G = \mathrm{S}(n) \times \mathrm{O}(m)$. Irreducible representations of G are just tensor products of constituents as they are absolutely irreducible. So, the knowledge of irreducible representations of $\mathrm{S}(n)$ and $\mathrm{O}(m)$ allows us to build an irreducible decomposition of the quotient algebra $\mathbb{R}[q_1, q_2, \ldots, q_n] / \mathcal{I}$ of the problem $(***)$. This allows to reduce complexity of problem $(***)$, because a large matrix becomes block-diagonal with small blocks.

New results in $(*)$ based on the solution of problem $(***)$ will be shown in the talk.

This work was supported by the Russian Foundation for Basic Research (project no. 15-01-02705) and by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1).

Orthogonal Latin squares in low dimensions

Matolcsi, Máté

The existence of complete sets of pairwise orthogonal Latin squares is equivalent to the existence of finite projective planes. We describe a possible method to prove non-existence or uniqueness of such objects in low dimensions (e.g. 6, 7, 8, 9 and possibly 10), based on discrete Fourier analysis and linear programming.

Joint work with Mihály Weiner.

Topics on Nörlund logarithmic means

Memić, Nacima

This work is a study of the maximal operator for a class of subsequences of Nörlund logarithmic means of Walsh-Fourier series. Almost everywhere convergence of $(t_{m_n} f)_n$ is obtained for every integrable function f. Besides, a divergence result holds for other classes of subsequences. On some unbounded structures a divergence result of Nörlund logarithmic means is proved for general Walsh-Fourier series.

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Moment inequalities for the maxima of partial sums in the probability theory with applications in the theory of orthogonal series

Móricz, Ferenc

Let $(X_k : k = 1, 2, \ldots)$ be a sequence of random variables. It is not assumed that these X_k's are mutually independent or that they are identically distributed. We set

$$S_{b,n} := \sum_{k=b+1}^{b+n} X_k \text{ and } M_{b,n} := \max_{1\le k\le n} |S_{b,k}|,$$

where $b \ge 0$ and $n \ge 1$ are integers. The object of our talk is to provide bounds on $\mathbf{E}(M_{b,n}^p)$ in terms of given bounds on $\mathbf{E}\,|S_{b,n}|^p$, where $p > 1$. We prove the generalized Rademacher-Menshov maximal moment inequality, the generalized Erdős-Stechkin maximal moment inequality, and the upper part of the Law of the iterated Logarithm.

On a potential theoretic minimax problem on the torus

Nagy, Béla

In this talk some new results will be presented based on the joint work with Bálint Farkas and Szilárd Gy. Révész. The results are available at the `http://arxiv.org/abs/1512.09169` webpage.

First, we recall some general results and notation on general potential theory (e.g. potentials, Chebyshev constants), and then some well known phenomena originating in interpolation theory (e.g. equioscillation property, sandwich property). Starting from a quite abstract context, we investigate a series of questions keeping generality as long as possible. We also cite some new results on minimax problems on the torus.

Then we discuss some applications: a result of Bojanov on algebraic polynomials with prescribed zero order having minimal sup-norm on interval. As an application of our general framework we show a generalization of Bojanov's result for generalized algebraic polynomials (GAP) and generalized trigonometric polynomials (GTP) as well.

Some properties of Marcinkiewicz means with respect to Walsh system

Nagy, Károly

The behaviour of the maximal operator of Marcinkiewicz means with respect to the Walsh and Walsh-Kaczmarz system was studied by a lot of authors [Wei01, Gog08, GGN09].

It was shown that the endpoint of the boundedness of this maximal operator is $p = 2/3$ [Gog06, GN09].

We gave additional details to the known results in the endpoint [Nag11, NT14a, NT14b, NT16].

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Some properties of Orlicz algebras on groups

Öztop, Serap

Let G be a locally compact group, Φ be a Young function, and denote by $L^{\Phi}(G)$ the associated Orlicz space. Orlicz spaces are precisely the $L^p(G)$ spaces for $1 \leq p < \infty$ and $\Phi(x) = x^p/p$. In [OoO15] and [OS17] we study the algebraic structure on $L^{\Phi}(G)$ with respect to convolution and twisted convolution, respectively. We continue our investigation on Orlicz spaces and try to find out under what condition they are dual Banach algebras. We apply our methods to compactly generated groups of polynomial growth and to a vast variety of cases.

This presentation is based on joint work with Ebrahim Samei, University of Saskatchewan, Canada.

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Slice regular Malmquist-Takenaka system in the quaternionic Hardy spaces

Pap, Margit

The present scientific contribution is dedicated to the 650th anniversary of the foundation of the University of Pécs, Hungary. The project has been supported by the European Union, co-financed by the European Social Fund EFOP-3.6.1.-16-2016-00004.

Stephen L. Alder in [Adl86] provides an introduction to the problem of formulating quantum field theories in quaternionic Hilbert space. This well-written treatise is a very significant contribution to theoretical physics. But the full power of quaternions would be even more important by using the quaternionic analysis.

The theory of slice regular functions, and the use of the complete orthonormal systems and series expansions in this space, can be useful tool for the quantum theory.

The theory of slice regular functions of a quaternionic variable (often simply called regular functions) was introduced recently in [GS06, GS07]. and represents a natural quaternionic counterpart of the theory of complex holomorphic functions.

In this paper we introduce a slice regular analogue of the Malmquist-Takenaka system. It is proved that, under certain restrictions regarding to the parameters of the system, they form a complete orthonormal system in the quaternionic slice regular Hardy spaces of the unit ball. The properties of associated projection operator are studied.

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Images of some subspaces of $L^2(\mathbb{R}^m)$ under Grushin and Hermite semigroup

Patra, Partha Sarathi

In this talk I will present the image characterization of a subspace of $L^2(\mathbb{R}^{n+1})$ under Grushin semigroup as direct sum of two weighted Bergman spaces. We define Hermite Sobolev space of positive order on $\mathbb{R}^n$, and find out the image of this space under Hermite semigroup using Caputo fractional derivative as weighted Bergman space. We use the connection of Grushin operator and parametrized Hermite operator extensively and with that help image of Grushin Sobolev space of positive order is characterized.

Lebesgue functions of rational interpolations of non-band-limited functions

Pilgermájer, Ákos

This paper concentrates on the Lebesgue functions of rational interpolation of non-band-limited continuous time signals.

Approximation based on sampling and interpolation are cornerstones of applied mathematics. In the last years rational interpolations has been in the focus of the investigations, because they have better approximation properties then the polynomial interpolations. The Whittaker-Kotelnikov-Shannon sampling theorem is for band-limited signals and requires the a priori knowledge of the band-width. In [EP14], [KPP14] new rational interpolation operators were developed for the transfer function of non-band-limited signals, which can be used also in cases when the band-width is not known a priori. The construction of these operators is based on the discrete orthogonality of the Malmquist-Takenaka systems. Combining these interpolations one can give exact interpolation on the real line for a large class of rational functions among them for the Runge test function. Our aim is to study the properties of the Lebesgue function of these rational interpolation operators.

Joint work with Margit Pap.

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Arctic curves beyond the arctic circle

Prause, István

Random surfaces arising from the dimer model exhibit limit shape formation. There is a sharp spatial separation between frozen facets and disordered liquid regions. The arctic circle of domino tilings of the Aztec diamond is a celebrated example of this. We study the geometry and parametrization of such arctic curves using Kenyon-Okounkov theory and the intrinsic complex structure on the liquid region. This complex structure is described by a quasilinear Beltrami equation which degenerates at the frozen boundary. Thus the equation can be used to detect the boundary and its properties.

The talk is based on joint work with K. Astala, E. Duse and X. Zhong.

Rational Fourier – Chebyshev series of some elementary functions

Rouba, Yauheni

In the present report we consider the following system of Chebyshev – Markov rational fractions with real nonnegative parameter a:

$$M_n(x) = \cos n \arccos\left(x\frac{\sqrt{1+a^2}}{\sqrt{1+a^2x^2}}\right), \quad x \in [-1,1]\,,\ n = 0,1,\dots\,.$$

It is orthogonal on the segment $[-1,1]$ with respect to the weight

$$\rho(x,a) = \frac{\sqrt{1+a^2}}{(1+a^2x^2)\sqrt{1-x^2}}, -1 < x < 1.$$

Let f be absolutely integrable with respect to the weight $\rho(x,a)$ function on the segment $[-1,1]$. We associate the Fourier series with respect to the system $\{M_n\}$ to such function f:

$$f(x) \sim \frac{c_0}{2} + \sum_{n=1}^{+\infty} c_n M_n(x),$$

with coefficients:

$$c_n = \frac{2}{\pi}\int_{-1}^{1} \rho(t,a) f(t) M_n(t)\,dt, \quad n = 0,1,\dots\,.$$

Theorem 1. *For the partial sums of this Fourier series the following equality holds*

$$S_n(x;f) = \frac{1-\alpha^4}{2\pi}\int_{-\pi}^{\pi} f(\cos v)\frac{\sin{(n+1/2)\lambda(u,v)}}{\sin\lambda(u,v)/2}\frac{dv}{1+2\alpha^2\cos 2v+\alpha^4},$$

where

$$\lambda(u,v)=\int\limits_u^v \frac{1-\alpha^4}{1+2\alpha^2\cos 2y+\alpha^4}\,dy,\qquad \alpha=\frac{\sqrt{1+a^2}-1}{a}.$$

Besides, $S_n(x,f)=p_n(x)/\sqrt{(1+a^2x^2)^n}$, *where* $p_n(x)$ *is an algebraic polynomial of degree* n *and* $S_n(x,1)\equiv 1$.

In this work we study the Fourier series with respect to the system $\{M_n\}$ of the function $|x|$. We found Fourier coefficients in the explicit form. Let's introduce the following deviations:

$$\varepsilon_{2n}(x,\alpha)=|x|-s_{2n}(|x|;x),\ x\in[-1,1]\,;\quad \varepsilon_{2n}(\alpha)=\|\varepsilon_{2n}(x,\alpha)\|_{C[-1,1]}.$$

Theorem 2. *The following inequalities hold:*

$$|\varepsilon_{2n}(x,\alpha)|\le\frac{2}{\pi}\int\limits_0^1\sqrt{\frac{1+2\alpha^2\cos 2u+\alpha^4}{1+2t^2\cos 2u+t^4}}\,\frac{1-t^2}{1-\alpha^2t^2}\,|\chi_{2n}^*(t)|\,dt, x=\cos u, \qquad (*)$$

$$\varepsilon_{2n}(\alpha)\le\frac{4}{\pi}\int\limits_0^1\left|\frac{t^2-\alpha^2}{1-\alpha^2t^2}\right|^n\frac{dt}{1+t^2}.$$

Inequality (*) is exact in the sense that if all the poles have even multiplicity then inequality (*) becomes equality for $x=0$ *and* $x=1$.

Theorem 3. *If* $\varepsilon_{2n}=\inf_\alpha\varepsilon_{2n}(\alpha)$, *then the following estimates hold*

$$\overline{\lim_{n\to\infty}}\frac{n^2}{\ln n}\varepsilon_{2n}=\frac{1}{\pi},$$

$$\inf_\alpha|\varepsilon_{2n}(x,\alpha)|\le\frac{2}{\pi\,|x|}\frac{\ln n}{n^3},\ x\in[-1,0)\cup(0,1],\ n>n_0.$$

Also, we are going to briefly discuss the other results on asymptotic estimates of elementary functions by trigonometric series.

On the product and quotient sets of rational numbers and integers

Shteinikov, Yurii

Let A,B be the subsets of rational numbers:

$$A,B\subseteq F_Q=\{\frac{r}{s}:1\le r,s\le Q\}$$

and let C,D be the subsets of the interval $[1,Q]$.

I my talk I am planning to present the following result.

Theorem

1. There exists an absolute constant $C>0$ such that

$$|AB|\ge|A||B|exp\Big\{(-C+o(1))\frac{\log Q}{\log\log Q}\Big\},Q\to\infty.$$

2. There exists an absolute constant $c>0$, such that

$$|C/D|\ge|C||D|exp\Big\{(-2\log 2+c+o(1))\frac{\log Q}{\log\log Q}\Big\},Q\to\infty.$$

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Almost everywhere convergence of sequences of (C,α) means of m-adic Fourier series of integrable functions

Simon, Ilona

The talk investigates convergence questions on compact totally disconnected groups of the m-adic integers. It is known that the Fejér means - with respect to the product system of normed coordinate functions of continuous irreducible unitary representations of the coordinate groups - of an integrable function on these groups converge to the function a.e. In this work we prove the above for (C,α) means.

Joint work with György Gát.

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On images of Dunkl-Sobolev spaces in $L^2(\mathbb{R}, |u|^{2r} e^{x^2} dx)$ under Schrödinger semigroup

Sivaramakrishnan C

This is a joint work with my guide Dr. D. Venku Naidu and co-guide Dr. D. Sukumar.

For $r > 0$, we consider the Dunkl Laplacian Δ_r and characterize the image of Sobolev space in $L^2(\mathbb{R}, |u|^{2r} e^{x^2} dx)$ under Schrödinger semigroup $e^{it\Delta_r}$ as weighted Bergman space up to norms equivalence.

Spectral synthesis on affine groups

Székelyhidi, László

The classical spectral synthesis theorem of Laurent Schwartz states that every continuous complex valued function on the real line can be uniformly approximated on compact sets by exponential polynomials which belong to its *variety*: to the smallest translation invariant linear space which is closed under uniform convergence on compact sets. This phenomena is called *spectral synthesis*. The concept can be generalized to more sophisticated situations, like locally compact topological groups, Abelian or non-Abelian. Recently several extensions have been proved on discrete Abelian groups. Unfortunately, due to counterexamples of D. I. Gurevich, a reasonable extension of Schwartz's result to several real variables is impossible. In this talk we offer a new approach which generalizes the concepts of spectral synthesis to Gelfand pairs. We show that this approach can successfully be applied on non-commutative locally compact groups, in particular, on affine groups of finite dimensional vector spaces. As a consequence, we present a reasonable generalization of Schwartz's result to functions in several variables.

Approximation by Poisson polynomials in Smirnov classes with variable exponent

Testici, Ahmet

In this talk we discuss the approximation properties of Poisson polynomials in the variable exponent Smirnov classes of analytic functions. The direct and inverse theorems of approximation theory and the appropriate constructive characterizations of some subclasses were obtained in [IT15]. Moreover the approximation properties of de la Valée-Poussin and Jackson means in these spaces were studied in [IT16]. The problems considered in this talk are the continuations of the studies mentioned in [IT15] and [IT16].

Joint work with Daniyal M. Israfilov.

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Doubling condition at the origin for non-negative positive definite functions

Tikhonov, Sergey

We will discuss upper and lower estimates as well as the asymptotic behavior of the sharp constant $C := C_n(U,V)$ in the doubling-type condition at the origin

$$\frac{1}{|V|}\int_V f(x)\,dx \le C\,\frac{1}{|U|}\int_U f(x)\,dx,$$

where $U, V \subset \mathbb{R}^n$ are 0-symmetric convex bodies and f is a non-negative positive definite function.

Joint work with Dmitry Gorbachev.

The boundedness of the L^1-norm of Walsh-Fejér kernels

Toledo, Rodolfo

The L^1-norm of kernels corresponding to operators defined on different orthonormal systems plays an important role for the convergence theory of orthogonal series. A good example is the theorem of Kolmogorov-Seliverstov-Plessner which states a sufficient condition for the almost everywhere convergence of orthogonal series by the use of Lebesgue functions. Many researchers dealt with the properties of the L^1-norm of kernels. For the trigonometric system it is important to note the results of Fejér and Gábor Szegő, the latter gave an explicit formula for Lebesgue constants. Fine estimates are obtained the most properties of Lebesgue constants with respect to the Walsh-Paley system.

Another relevant operators in convergence theory are the Fejér means which are the arithmetical means of the partial sums of Fourier series. In case of the trigonometric system the L^1-norm of Fejér kernels is constant, so it does not depend on the index of the kernels. For Walsh-Paley system that kernels are very distinct, but they are also uniformly bounded, and this is the only known property so far. The aim of my talk is to show my results in the study of the L^1-norm of Fejér kernels with respect to the Walsh-Paley system. I established an iteration for the L^1-norm of Walsh-Fejér kernels. I use this iteration to prove some properties of this sequence, including that its supremum is exactly equal to $\frac{17}{15}$.

One-sided mean approximation on the Euclidean sphere to the characteristic function of a spherical layer by algebraic polynomials

Torgashova, Anastasiya

Let $\mathbb{S}^{m-1}$ be the unit sphere of the Euclidean space $\mathbb{R}^m$ of dimension $m \geq 3$. The set

$$\mathbb{G}(a, b) = \{x = (x_1, x_2, \ldots, x_m) \in \mathbb{S}^{m-1} \colon a \leq x_m \leq b\}$$

with $-1 \leq a < b \leq 1$ is called a spherical layer. In the case $a = -1$ or $b = 1$, the layer $\mathbb{G}(a, b)$ is the spherical cap. We will discuss the problem of one-sided approximation in the space $L(\mathbb{S}^{m-1})$ to the characteristic function of a spherical layer $\mathbb{G}(a, b)$ by the set $\mathcal{P}_{n,m}$ of algebraic polynomials of given degree n in m variables.

This problem reduces to the one-dimensional problem of one-sided approximation on the interval $(-1, 1)$ in the space $L_1^{\phi}(-1, 1)$ ñ with the ultraspherical weight $\phi(t) = (1 - t^2)^{\alpha}$, $\alpha = (m - 3)/2$, to the characteristic function of the interval $[a, b]$. The latter problem will be discussed in a space of functions integrable over $(-1, 1)$ with a more general, not necessarily ultraspherical, weight.

The talk is based on joint research with M.Deikalova.

This work was supported by the Russian Foundation for Basic Research (project no. 15-01-02705) and by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-9356.2016.1).

Ramanujan-Fourier expansions of arithmetic functions of several variables

Tóth, László

Let $c_q(n)$ denote the Ramanujan sum, defined as the sum of n-th powers of the primitive q-th roots of unity. Let $\sigma(n)$ and $\tau(n)$ be the sum and the number of divisors of n, respectively. According to Ramanujan's classical identities, for every fixed $n \in \mathbf{N}$,

$$\frac{\sigma(n)}{n} = \zeta(2) \sum_{q=1}^{\infty} \frac{c_q(n)}{q^2}, \qquad \tau(n) = -\sum_{q=1}^{\infty} \frac{\log q}{q} c_q(n),$$

where ζ is the Riemann zeta function.

We discuss the expansions of certain arithmetic functions of several variables with respect to the Ramanujan sums $c_q(n)$ and their unitary analogues $c_q^*(n)$. We show, among others, that the series

$$\frac{\sigma(\gcd(n_1, \ldots, n_k))}{\gcd(n_1, \ldots, n_k)} = \zeta(k+1) \sum_{q_1, \ldots, q_k = 1}^{\infty} \frac{c_{q_1}(n_1) \cdots c_{q_k}(n_k)}{\operatorname{lcm}(q_1, \ldots, q_k)^{k+1}} \quad (k \geq 1),$$

$$\tau(\gcd(n_1, \ldots, n_k)) = \zeta(k) \sum_{q_1, \ldots, q_k = 1}^{\infty} \frac{c_{q_1}(n_1) \cdots c_{q_k}(n_k)}{\operatorname{lcm}(q_1, \ldots, q_k)^{k}} \quad (k \geq 2),$$

are absolutely convergent for every fixed $n_1, \ldots, n_k \in \mathbb{N}$. Our results and proofs generalize and simplify those of Ushiroya [Ush16].

References

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A short proof of a duality theorem and another application of an intersection formula on dual cones

Virosztek, Dániel

We give a succinct proof of a duality theorem obtained by Révész in 1991 [1] which concerns extremal quantities related to trigonometric polynomials. The key tool of our new proof is an intersection formula on dual cones in real Banach spaces. We show another application of this intersection formula which is related to the integral estimates of non-negative positive definite functions.

References

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Convergence of rectangular summability and Lebesgue points of higher dimensional Fourier transforms

Weisz, Ferenc

Three types of rectangular summability of higher dimensional Fourier transforms are investigated with the help of an integrable function θ, the unrestricted summability, the summability over a cone and over a cone-like set. We introduce the concept of different Lebesgue points and show that almost every point is a Lebesgue point of f from the Wiener amalgam space $W(L_1, \ell_\infty)(\mathbb{R}^d)$. We give three generalizations of the well known Lebesgue's theorem for the summability of higher dimensional Fourier transforms. More exactly, under some conditions on θ we show that the different types of summability means of a function $f \in W(L_1, \ell_\infty)(\mathbb{R}^d) \supset L_1(\mathbb{R}^d)$ converge to f at each Lebesgue point.

Cultural and technical information

1 Technical Information

1) 24th of August: The participants are expected to arrive until 19:00 of August 24th and to leave in the morning of August 31st. The registration will take place on the first floor of the workshop venue, by the entrance of the big lecture room. The welcome party will be held on the 24th of August between 20:00-22:30. Please arrive by 19:00 to the Headquarter of the Academy to register or take the workshop bus at 19:00 starting near the Koch Valeria Dormitory, at the Car Park of Hotel Makár. Those staying at Koch Dormitory can take the workshop bus back to the Dorm somewhere between 22-23:00 (to be announced later) starting from the car park of the Headquarter.

2) 31st of August: The participants are expected to leave in the morning of August 31st or take the workshop shuttle at 15:30. There are some free places in the shuttle to Budapest Airport (not passing through Budapest) at 9:00 starting from Koch Dormitory and stopping at the Headquarter and Centrum ApartmanHotel. There are free places also on the workshop bus starting at 15:30 from Koch Dormitory, stopping at the Headquarter and Budapest Metro station Kőbánya-Kispest. You can sign up for these.

3) The scientific program and coffee breaks will take place mostly at the Headquarter of the Hungarian Academy of Sciences named „PAB Székház" (Jurisics Miklós street 44). Exceptionally, on Monday, the 28th August, the workshop will be held in the Conference Room at the University of Pécs (Vargha Damján Konferenciaterem) (Ifjúság street 6.) A laptop, projector small white board will be arranged. There will be talks of 45 and 25 minutes long, which means a duration of 40/20 minutes for lecture and 5 minutes for discussion.

4) Accommodation. There is wifi on each of these places. The breakfasts are continental.

Headquarter of the Hungarian Academy of Sciences named „PAB Székház" (44 Jurisics Miklós street). Breakfast included.

Koch Valeria Student dormitory (13 Mikes Kelemen street). You were asked in the questionnaire whether you wish breakfast or not, because the organizers had to order it. It is at the university's canteen/dining room, named Pacsirta Restaurant between 7:30-8:15. At 8:30 a bus takes you from the Pacsirta street Car Park to the Headquarter. Except Sunday, as the University is closed then and the Restaurant gives cold packets for breakfast. On Sunday the bus starts from the Car Park of Hotel Makár at 9:00, which is near to Koch Dormitory. Punctuality at the bus departures is strictly required.

Centrum ApartmanHotel (11 Damjanich street). The wifi code can be found at the reception. Breakfast included.

5) **Social events**: We offer several social events: a welcome dinner, an excursion with guided sightseeing tour and wine tasting in Villány, a banquet and an optional afternoon for free time, museums or spa. If your accompanying person(s) also take part, then indicate it in the questionnaire.

- Excursion on the 27th of August: guided sightseeing tour in Pécs including some museums, churches in the morning. After lunch we visit the Széchenyi Square, the Zsolnay Cultural Quarter. In the evening we will visit a wine cellar in the famous wine-village Villány. See details below. Accompanying persons are welcome.

- The banquet will be held on the 29th of August from 17:00, at the Headquarter of Academia, beginning shortly after the scientific program. Accompanying persons are welcome.

- Museum afternoon/spa afternoon/free time on the 30th of August. We leave after lunch at 14:00 from the entrance of the Headquarter. Both programs end at 18:00. This time is just enough for 3-4 museums, but it is quite short for traveling to Harkány Spa. Entrance fees for this afternoon are your own expenses and a registration for museums is recommended until the morning of 28th. Museums: Cella Septichora, Csontváry Museum, Vasarelly Museum, (and maybe Zsolnay Museum). The recommended spa is at Harkány, we only match those who are interested in it and ask you to organize your own trip and take the bus to Harkány or ask somebody who has a car to take you there. (http://www.harkanyfurdo.hu/en)

- Physical exercise on the 26th of August, 17:15-18:15, after the scientific program, hopefully outdoors in the back garden of the Headquarter. It will be canceled in rainy weather. This program is suitable also for participants in their sixties, it will be a mobilization of small intensity but sportswear is needed. (On demand this can be repeated on the 30th between 20:30-21:30. Also, a limited number of people (attendees over 40 years old) is invited to attend my Senior class on Monday, the 28th, at 20:15 for free in the air-conditioned Gyémánt Fitness Club, 26 Nagy Jenő street. Registration is needed at Ilona Simon.)

6) Publication: You are kindly invited to submit your paper BY THE END OF SEPTEMBER, if possible, to our journals in Hungary. Papers submitted by the conference participants and accepted by referees will be published in either no. 1, vol. 44 (2018) of Analysis Mathematica, or in no. 2, vol. 26 (2017) of Mathematica Pannonica, both regular issues, but dedicated to the Workshop. When submitting your paper, please indicate which journal you would like to publish The submitted papers will undergo a regular refereeing process. For more information on these journals see their home pages at
`http://www.springer.com/mathematics/analysis/journal/10476` and at
`http://mathematica-pannonica.ttk.pte.hu/index.htm` respectively. Note that both journals are reviewed from cover to cover by Mathematical Reviews / Math.Sci.Net and Zentralblatt, and Analysis Mathematica has IF 0.325 and is also in WoS.

2 Excursion details for 27th of August

Punctuality at the meeting points and bus departures is strictly required.

The bus starts at 9:00 from Koch Dormitory (Car Park of Hotel Makár) and at 9:10 from the Headquarters and takes the guests to the bastion Barbakán. Guests staying at Centrum ApartmanHotel can walk within 3 minutes to Barbakán.

Beginning from 9:15 we will have a walk in the city center. The guide will tell us stories and historical details on the bastion Barbakán, the city wall, the Dom Square, Bishopric Palace, basilica, the Museum street, the Széchenyi Square, the UNESCO world Heritage Cella Septichora Center, Turkish sites, love locks.

At Szent István square we will visit the Episcopal Palace with its Mediterranean Garden and the Central Church of the Bishopric of the Diocese of Pécs. We will climb up to the tower and enjoy organ music.

12,15-13,30 Lunch at Pezsgőház Restaurant.

13:30-14:30 At Széchenyi Square we will visit the Mosque of Pasha Gázi Kászim.

14:30 We travel by bus to the Zsolnay Cultural Quarter to enjoy a guided tour and some free time there.

15-16 Free time at Zsolnay Cultural Quarter. You can visit the Museum „The Golden Age of Zsolnay – Gyugyi Collection" as an optional program on your expenses. Alternatively there are some shops, like the Chocolate Shop (http://www.csokolada.hu/), the show of production of sweets beginning at 15:00 in Nosztalgia Cukorka Látványmanufaktúra.

16:00 The bus starts for the famous village Villány. Wine-tasting in a wine-cellar.

21:00 We travel back to Pécs.

3 Dinners

Remember, that your dinners are already arranged for the 24th and 29th.

Nearby Restaurants to the Headquarter:

1) Salibár, http://www.salibar.hu/ujsalata/, Monday-Saturday:12:00-22:00, Sunday: 12:00-19:00. Tel: 0036-72-212-926, 12. Bartók Béla street.

2) The restaurant of Hotel Szinbád. http://www.szinbadhotel.hu

Nearby Restaurant to Koch Dormitory:

3) Café Paulus, http://pauluscafe.hu/, 4. Ifjúság street. Tel: between hours 8-19: 0036(30) 383 2996, after 19:00: 0036 (72) 503 636, Opening hours: Monday- Friday: 9:00 – 0:00, Saturday: 11:30 – 0:00, Sunday closed.

Other recommended restaurants:

4) The very best gourmet restaurant is **Susogó Borvendéglő** - according to a gastronomy-guru colleague. 54. Szőlő street. Open 17-22, Sunday and Monday closed. Tel: 0036-70-291 0403, https://www.facebook.com/susogopecs/. It is within 2,1 km, 6 minutes by car and 30 minutes on foot on some hilly path.

5) **Tettye Restaurant** – for Hungarian and Swabian specialties. Opening hours: Monday - Sunday: 11.00 - 23.00, 4. Tettye square. Tel: 003672/532-788. http://www.tettye.hu/main. Tettye Restaurant can be found on the southern side of the mountain Mecsek in a picturesque environment. The menu offers a generous variety of fish, poultry with crispy vegetables and smooth creamy sauces, but you can also find famous Hungarian originals like the pörkölt, goulash, the paprika stew, meat roasted on a spit, or you can taste typical Swabian specialties like the feast soup, cabbage and beans, and the Swabian casserole with Bohemian bread dumplings. You can travel to Tettye Restaurant from the city center by bus 33. The line has two bus stops directly in front of the restaurant, going in both directions. If you are with a car you can go on Hunyadi János street and start driving up the hill until you get to Pálos church (in front of which you see the Vasarely statue). At the intersection in font of the church you go to the right and follow the Magaslati road wich leads to Tettye square. Before going home you can have a walk at Tettye chapel, cross, ruins, there is a wonderful view on the city.

6) Elefántos Étterem és Pizzéria is the best for Pizza and Italian cuisine in town. Ristorante and Pizzeria All'Elefante, http://elefantos.hu/ 6. Jókai street, Tel: 0036-72- 216 055, Opening hours: 12:00–23:00.

7) For tapas we recommend "Eleven bor and tapas" in the City Center, 11. Jókai square. Opening hours: Wednesday-Saturday 17:00-23:00, Tel: 0036-20-262 6750, https://www.facebook.com/elevenborestapas/

8) Lezser Bárkonyha, 18. Széchenyi square. 0036 (70) 537 6733, Open: 9:00–24:00

9) Other restaurants on Király Street in the City Center.

4 About Pécs

Pécs is the fifth largest city of Hungary, located on the slopes of the Mecsek mountains in the south-west of the country, close to its border with Croatia. The city Sopianae (this is the ancient name of the city) was founded by Romans at the beginning of the 2nd century, in an area peopled by Celts and Pannonitribes. By the 4th century, it became a significant early Christian center. The early Christian necropolis is from this era which became a UNESCO World Heritage Site in December 2000.

Pécs is the seat of the Roman Catholic Diocese of Pécs. Its episcopate was founded in 1009 by King Stephen I, and the first university in Hungary was founded in Pécs in 1367 by King Louis I the Great. (The largest university still resides in Pécs with about 34,000 students). Pécs was formed into one of the cultural and arts center of the country by bishop Janus Pannonius, great humanist poet. Pécs has a rich heritage from the age of a 150-year-long Ottoman occupation, like the mosque of Pasha Qasim the Victorious on Széchenyi Square or the mosque of Pasha Jakováli Haszan on the Kórház Square.

Pécs always was a multicultural city where Hungarians, Croatians and Swabians still live in peace together in economic and cultural polarity. In 1998 Pécs was given the UNESCO prize Cities for peace for maintaining the cultures of the minorities, and also for its tolerant and helping attitude toward refugees of the Yugoslav Wars. Pécs was nominated as third, and second Livable city in the category of cities between 75,000 and 200,000 inhabitants. In 2010, Pécs was selected to be the European Capital of Culture sharing the title together with Essen and Istanbul.

Name: The earliest name for the territory was its Roman name of Sopianæ. The name possibly comes from the plural of the Celtic sop meaning "marsh". Contrary to the popular belief, the name did not signify a single city (Sopianae: plural), and there are no traces of an encircling wall from the early Roman era, only from the 4th century. The medieval city was first mentioned in 871 under the name Quinque Basilicae ("five cathedrals".) The name refers to the fact that when constructing the churches of the city, the builders used material from five old Christian chapels. The German name of the city is Fünfkirchen. The name Pécs appears in documents in 1235 in the word Pechyut.

Geography: It is bordered by Mecsek from the north, and a plain from the south. Pécs has a significant mining past. Mecsek dolomitic water is famous for its high density of minerals at constant poise. It has a very favorable climate by the border of a still flourishing woody area. During the hot summer nights a cooling air streams down from Mecsek to clean the air of the city.

History of Pécs:

The area has been inhabited since ancient times, with the oldest archaeological findings being 6,000 years old. Before the Roman era the place was inhabited by Celts. When Western Hungary was a province of the Roman Empire (named Pannonia), the Romans founded several wine-producing colonies under the collective name of Sopianae where Pécs now stands, in the early 2nd century.

The centre of Sopianae was at the place of the present Postal Palace. Some parts of the Roman aqueduct are still visible. In the first half of the 4th century, Sopianae became an important Christian city. The first Christian cemeteries, dating back to this age, are inscribed on the World Heritage List. By the end of the century, Roman rule weakened in the area, mostly due to attacks by Barbarians and Huns.

When Charlemagne arrived in the area in the Middle Ages, it was ruled by Avars. Charlemagne, after conquering the area, annexed it to the Holy Roman Empire. It belonged to the Diocese of Salzburg.

After then the Hungarians conquered the Carpathian Basin, they retained a semi-nomadic lifestyle changing pastures between winter and summer. Árpád's winter quarters were perhaps in Pécs, clearly after his occupation of Pannonia in 900. Later, Pécs became an important religious centre and episcopal seat. In 1064, when King Solomon made peace with his cousin, the later King Géza I, they celebrated Easter in Pécs. Shortly after, the cathedral burnt down. The cathedral that stands today was built after this, in the 11th century.

Several religious orders settled down in Pécs. The Benedictine order was the first in 1076. In 1181, there was already a hospital in the city. The first Dominican monastery of the country was built in Pécs in 1238. King Louis the Great founded a university in Pécs in 1367, it was the first university in Hungary. The founding document is almost word for word identical with that of the University of Vienna, stating that the university has the right to teach all arts and sciences, with the exception of theology. (today Pécs has a Catholic Theology.) In 1459, Janus Pannonius, the most important medieval poet of Hungary became the bishop of Pécs. He strengthened the cultural importance of Pécs.

Pécs under Ottoman rule:

After the Battle of Mohács (1526) in which the invading Ottoman army defeated the armies of King Louis II, the armies of Suleiman occupied large teritories of Hungary. Not only was a large part of the country occupied by Ottomans, the public opinion of who should be the king of Hungary was divided, too. One party supported Ferdinand of Habsburg, the other party crowned John Szapolyai in Székesfehérvár. The citizens of Pécs supported Emperor Ferdinand, but the rest of Baranya county supported King John. In the summer of 1527 Ferdinand defeated the armies of John Szapolyai and was crowned king on November 3. Ferdinand favoured the city because of their support, and exempted Pécs from paying taxes. Pécs was rebuilt and fortified.

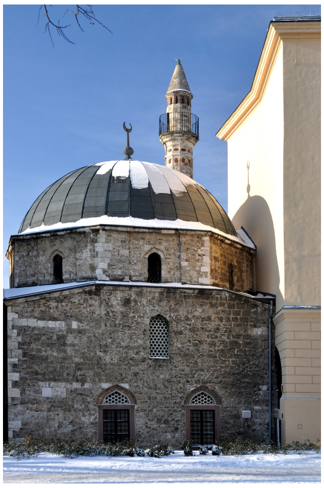

Figure 1: Mosque Jakovali Haszan, photo by György Mánfai

In 1529, the Ottomans captured Pécs again, and went on a campaign against Vienna. The Ottomans made Pécs to accept King John (who was allied with them) as their ruler. John died in 1540. In 1541, the Ottomans occupied the castle of Buda, and ordered Isabella, the widow of John to give Pécs to them, since the city was of strategic importance. The citizens of Pécs defended the city against the Ottomans, and swore loyalty to Ferdinand. The emperor helped the city and defended it from further Ottoman attacks, but his advisers persuaded him into focusing more on the cities of Székesfehérvár and Esztergom instead of Pécs. Pécs was preparing for the siege, but a day before, Flemish and Walloon mercenaries fled from the city, and raided the nearby lands. The next day in June 1543 the Bishop himself went to the Ottomans with the keys of the city.

After occupying the city, the Ottomans fortified it and turned it into a real Ottoman city. The Christian churches were turned into mosques; Turkish baths and minarets were built, Qur'an schools were founded, there was a bazaar in place of the market. In 1664, Croat-Hungarian nobleman Nicholas Zrínyi arrived in Pécs, with his army. Since the city was well into the Ottoman territories, they knew that even if the occupy it, they could not keep it for long, so they planned only to pillage it. They ravaged and burned the city but could not occupy the castle. The medieval Pécs was destroyed, except the wall encircling the historical city, a single bastion (Barbakán), the network of tunnels and catacombs beneath the city, parts of which are closed down, other parts are in possession of the famous Litke champagne factory, and can be visited today. Several Turkish artifacts also survived, namely three mosques, two minarets, remnants of a bath over the ancient Christian tombs near the cathedral, and several houses, one even with a stone cannonball embedded in the wall.

After the castle of Buda was wrested from Ottoman rule in 1686, the armies went to capture the rest of Pécs. The advance guards could break into the city and pillaged it. The Ottomans saw that they could not hold the city, and burnt it, and withdrew into the castle. The army led by Louis of Baden occupied the city on 14 October and destroyed the aqueduct leading to the castle. The Ottomans had no other choice but to surrender, which they did on 22 October.

Slowly the city started to prosper again, but in the 1690s two plague epidemics claimed many lives. In 1688 German settlers arrived. Only about one quarter of the city's population was Hungarian, the others were Germans or Southern Slavs. According to 1698 data, South Slavs comprised more than a half of the population of the town. Because Hungarians were only a minority of the population, Pécs did not support the revolution against Habsburg rule led by Francis II Rákóczi, and his armies pillaged the city in 1704.

Pécs in early-modern times and during the 19th century

A more peaceful era started after 1710. Industry, trade and viticulture prospered, manufactures were founded, a new city hall was built. In 1777, Queen Maria Theresa quickly elevated Pécs to free royal town status.The industry developed a lot in the second half of the 19th century. Some of the manufactures were nationally famous. The iron and paper factories were among the most modern ones of the age. Coal mining was relevant. A sugar factory and beer manufactures were built, too. The city had 14,616 residents.

During the revolution in 1848–49, Pécs was occupied by Croatian armies for a short time, but it was freed from them by Habsburg armies in January 1849. After the Austro-Hungarian Compromise of 1867 Pécs developed. At the end of World War I, Baranya county was occupied by Serbian troops, and it was not until August 1921 that Pécs could be sure that it remains part of Hungary. The University of Pozsony/Bratislava (Slovakia) was moved to Pécs after Hungary lost Bratislava according to the Treaty of Trianon.

During World War II, Pécs suffered only minor damages. After the war, development became fast again, and the city grew, absorbing several nearby towns. In the 1980s, Pécs already had 180,000 inhabitants.

After the end of Socialist era, after 1990, Pécs and its county, were hit hard by the changes, the unemployment rate was high, the mines and several factories were closed, and the war in neighboring Yugoslavia in the 1990s affected the tourism.

A good example of the city's history and interesting past can be seen in the main square, where the Gazi Kasim Mosque still stands, and, although consecrated as a church following the retreat of the Ottoman Turks centuries ago, the crescent moon of Islam and cross of Christianity are still visible on the cupola. Indeed, Pécs is the richest town in Hungary in terms of Turkish architecture, with the ruins of Memi

Pasa's Baths and the mausoleum of miracle worker Idris Baba, and other remains.

Pécs has an internationally famous porcelain factory. One special type of Zsolnay Porcelain has a special iridescent surface — called "eozin". Some of the walls of buildings on the Main Square is decorated with Zsolnay Porcelain tiles, as well as the walls and roofs of several public buildings. The Pécsi Sörfőzde (Pécs Brewery) is one of the four main Hungarian breweries, and produces a special beer, which is not strained before bottling. Pécs is also known for its leather-working industry.

Széchenyi Square

This is the main square in the historical centre of Pécs, Hungary. In the Middle Ages it served as the market place of the town with the city hall and the parish church. The square is full of monuments. Its main attractions are the Mosque of Pasha Qasim, the City Hall, the County Hall, the Nádor hotel, the Zsolnay well, the Fatebenefratelli Church, the Trinity statue and the brass statue of János Hunyadi on horse back.

Figure 2: Mosque Gázi Kászim, today catholic church, photo by György Mánfai

The central building of Széchenyi square is **the Mosque of Pasha Qasim the Victorious**, one of the symbols of Pécs. In the Middle Ages, the Gothic Saint Bertalan parish church stood in the middle of the city centre. The Saint Bertalan church was built in the age of Árpád. After the city was taken over by the Turks, the mosque of pasha Qasim was built from the rocks of Saint Bertalan. In the inner wall, by the current main entrance, there is a mihrab that indicates the qibla; that is, the direction of the Kaaba in Mecca. The building took its current form during the reconstruction in 1939. Saint Bertalan bell is next to the mosque.

The statue of János Hunyadi on horse back is set on a simple stone pedestal and was inaugurated in 1956, the 500th anniversary of the death of János Hunyadi. The Trinity statue in the middle of the square reached completion in 1714 to commemorate the secession of an earlier plague. On the southern part you find a church with Zsolnay well in front. Let us have a look at the eosin well of Zsolnay. On the four side of the four meter high well water gurgles through heads of oxen into an arc shaped basin. The well, donated by Miklós Zsolnay is decorated by the coats of arms of the city and the family. The Fatebenefratelli Church was built between 1727-1731. Its facade is eclectic. The first floor is decorated by through-gutter. Its inlaid, wooden altars have exceptional artistic value. The baroque Italian main altar painting depicts the martyrdom of Saint Sebastian.

On the western part you can find the Pécs **County house**. Its facade looking to the square is much more decorated than other buildings on the square. Its art pottery decors were made in Zsolnay factory. Lajos Nagy Gymnasium is located to the north from the county hall building. It was built between 1716 and 1726 by the Jesuits using the thumb stones of the Turkish cemetery on Kórház square. It was owned by the Cistercians from the beginning of the 20th century until the order ended. Frescoes in the stairway depict the foundation and structure of the first university of Pécs. The northern part of the square is partly made up by one of the buildings of Janus Pannonius Museum. Its main front has copf style and its side has classic style. There are two beautiful iron mashed gate from the 19th century in its doorway.

On the easter part we find the City Hall. The first city hall was built after the Ottoman occupation of Hungary in 1710, and the second one between 1831-1832. During the reconstruction in 1907, it received its current form with baroque elements on the front. The Nádor Hotel was built in 1846 which was demolished in 1902 to build a larger one in its place. By the end of the 1980s the facade of the hotel building was seriously damaged. After being closed for over fifteen years its facade was reconstructed in 2005 and an underground parking place was constructed. A gallery was opened in the building in 2009, but the hotel has not opened yet.

5 Recommended museums

4TH CENTURY EARLY CHRISTIAN BURIAL SITES: Cella Sepitchora Visitor Centre and Early Christian Mausoleum

OPENING HOURS: Tuesday – Sunday: 10:00–18:00, Monday: closed. Address: Szent István Square. ENTRANCE FEES: Full price: 1.700 HUF, Student / Teacher / Pensioner: 900 HUF In case of groups of 10 or more they offer a 15 percent discount from full price entrance ticket. A recommended program for the afternoon of 30th August.

Sopianae, predecessor of Pécs in the Roman times had its late Roman Paleochristian cemetery included in the UNESCO World Heritage list in the year 2000. In their architecture and wall-paintings the excavated finds present the Early Christian burial architecture and art of the Northern and Western provinces of the Roman Empire.

The several thousands of tombs, the numerous burial chambers, the burial chapel and the greater cemetery buildings (the mausoleum, Cella Trichora and the Cella Septichora) refer to a religious centre and a Christian community with a high number of members. The known Early Christian monuments that lie under the Dóm Square have been united with the newly excavated parts to make one building complex people can walk around. The result is the Cella Septichora Visitor Center today.

The Cella Septichora is actually one of the largest buildings (the burial site with seven apses) which we will see from outside on our excursion on Sunday, but visiting the whole center is worth the time as there are 2 painted chambers and one painted mausoleum, forming a unique monument from this era.

The uniqueness of the Early Christian cemetery of Sopianae is that here a high number of buildings were concentrated. These are smaller family burial chambers and larger community sepulchral vaults, cemetery buildings. A part of these are decorated; inside they have pictures of biblical scenes and Early Christian symbols that further raise the uniqueness and the universal cultural values of these monuments.

The sepulchral chamber served with burial sites mainly for richer families. The building has two parts: under the ground is the crypt; the burial chamber itself where the deceased were placed in brick tombs or seldom in sarcophagus's. Above this was the 'memoria' or in other words the mausoleum built. These two-part buildings had dual tasks: they served as building sites and as venues for ceremonials.

The **Peter and Paul burial chamber** was recovered in 1782. On the Northern wall of the chamber visitors can see the portrayal of Peter and Paul apostles who point at the Christogram in an extolling posture. The Christogram is a symbol of Jesus consisting of the Greek X (khi) and P (rho) letters that give the first two letters of the Greek name of Christ: (KhRisztosz); hence the name the monogram of Christ, Christogram.

On the ceiling of the burial chamber visitors can marvel at one outstanding example of 4th Century Early Christian wall painting. The painting symbolizing paradise has a rich plant (grapes) and animal (peacock) ornament and the pictures of four young people - whose characters are unknown - cased in medallions. Here you can find 6 pictures with biblical theme.

The **Korsós 'Pitcher' burial chamber** was also discovered during the establishment of the extensive Pécs cellar system in the 18th century. The chamber got its name from the pitcher and glass symbols that are found in the box in its northern wall. Plant and geometric decorations can be seen on the walls and the rail motif probably refers to the garden of paradise.

VASARELY MUSEUM

Address: 3 Káptalan Street. Opening hours: Tuesday-Sunday 10 a.m - 18 p.m. Mondays closed. This is a recommended program for the afternoon of 30th.

The Vasarely Museum is one of Pécs' most popular collections.

Figure 3: Picture by Vasarely

Vasarely was born in Pécs under the name Vásárhelyi Győző, grew up in Pöstyén (now in Slovakia) and Budapest and settled in Paris in 1930. He died age 90 in Paris in 1997. Vasarely produced art and sculpture using optical illusion. This Hungarian–French artist is widely accepted as a "grandfather" and leader of the op art movement. His work entitled Zebra, created in the 1930s, is considered by some to be one of the earliest examples of op art. (You can find this piece of art in the museum.) He developed his style of geometric abstract art, working in various materials but using a minimal number of forms and colors. Vasarely experimented with textural effects, perspective, shadow and light. Beginning from 1947 he started to develop geometric abstract art (optical art) and finally, Vasarely found his own style. He worked on the problem of empty and filled spaces on a flat surface as well as the stereoscopic view.

(**Op art**, short for optical art, is a style of visual art that uses optical illusions. Op art works are abstract, with many better known pieces created in black and white. Typically, they give the viewer the impression of movement, hidden images, flashing and vibrating patterns, or of swelling or warping.)

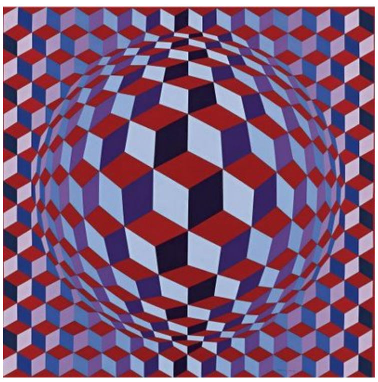

Figure 4: Picture by Vasarely

Kinetic images, black-white photographies: He then created kinetic images in black-white which create dynamic, moving impressions depending on the viewpoint. In the black-white period he combined the frames into a single pane by transposing photographies in two colors. Building on the research of constructivist and Bauhaus pioneers, he postulated that visual kinetics (plastique cinétique) relied on the perception of the viewer who is considered the sole creator, playing with optical illusions.

Folklore planétaire, permutations and serial art: Permutations of geometric forms are cut out of a colored square and rearranged. He worked with a strictly defined palette of colors and forms (three reds, three greens, three blues, two violets, two yellows, black, white, gray; three circles, two squares, two rhomboids, two long rectangles, one triangle, two dissected circles, six ellipses) which he later enlarged and numbered. Out of this plastic alphabet, he started serial art, an endless permutation of forms and colors worked out by his assistants. (The creative process is produced by standardized tools and impersonal actors which questions the uniqueness of a work of art.)

The Tribute to the hexagon series consists of endless transformations of indentations and relief adding color variations, creating a perpetual mobile of optical illusion.

His works can be seen at Centre Pompidou in Paris and the Vasarely Museum located at his birthplace in Pécs, and in the second Hungarian Vasarely museum, which was established in Zichy Palace in Budapest with more than 400 works. A new Vasarely exhibit was mounted in Paris at Musee en Herbe. There is a Vasarely Museum in Gordes Palace, Vaucluse, France (closed) and Fondation Vasarely in Aix-en-Provence, France.

CSONTVÁRY MUSEUM

Address: 11. Janus Pannonius street. Opening hours: Tuesday-Sunday 10 a.m - 18 p.m. Closed on Mondays. A recommended program for the afternoon of 30th.

Tivadar Csontváry Kosztka (1853 –1919) was a Hungarian painter who was part of the avant-garde movement of the early twentieth century. Working mostly in Budapest, he was one of the first Hungarian painters to become known in Europe. His works are held by the Hungarian National Gallery in Budapest and the Csontváry Museum in Pécs, among other institutions and private collectors.

Csontváry was a pharmacist until his twenties. On a hot sunny afternoon when he was 27 years old, he had a mystic vision. He heard a voice saying, "You are going to be the greatest painter of the world, greater than Raphael." He took journeys around Europe, visited the galleries of the Vatican, and returned to Hungary to earn money for his journeys by working as an apothecary. From 1890, he traveled around the world. He visited Paris, the Mediterraneum (Dalmatia, Italy, Greece), North Africa and the Middle East (Lebanon, Palestine, Egypt, Syria) and painted pictures. Often his pictures are very large, several meters (yards) wide and height is not unusual.

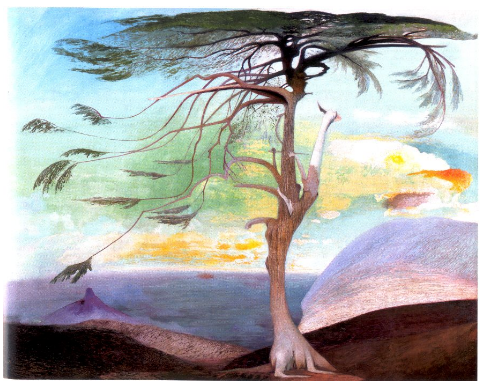

Figure 5: The lonely Cedar

Most of the critics in Western Europe recognized his abilities, art and congeniality, but in the Kingdom of Hungary during his life, he was considered to be an eccentric crank for several reasons, e. g. for his vegetarianism, anti-alcoholism, anti-smoking, pacifism, and his cloudy, prophetic writings and pamphlets about his life, genius and religious philosophy. Some of his biographists considered this as a latent, but increasingly disruptive schizophrenia. Although he was later acclaimed, during his lifetime Csontváry found little understanding for his visionary, expressionistic style. A loner by nature, his “failure” impaired his creative power.

He painted more than one hundred pictures, the most famous and emblematic of which is probably The Lonely Cedar (Magányos cédrus). His art connects with post-impressionism and expressionism, but he was an autodidact and cannot be classified into one style. He identified himself as a "sunway"-painter, a term which he created.

The Csontváry Museum in Pécs, Hungary, was founded in his honor and holds many of his works.

"I, Tivadar Kosztka, who gave up his prime of youth for the rebirth of the world, accepting the call of the invisible Spirit, had a regular civil job, comfort, wealth then (...) Going to Paris in 1907, I oppositely stood alone in front of millions with only the result of the divine providence, and I beat the vanity of the world hollow, but I haven't killed 10 million people, only sobered them, I haven't made commercials from things, because I didn't care for the pedlar's press; I retired from the world instead, going to the top of the Lebanons, and I painted cedars." T., Cs. K.: The Positivum.

ZSOLNAY PORCELAIN. ZSOLNAY MUSEUMS

The Zsolnay Porcelain Manufactory is a Hungarian manufacturer of porcelain, tiles, and stoneware. The company introduced the eosin glazing process and pyrogranite ceramics.

Figure 6: Zsolnay Porcelain well

The Zsolnay factory was established by Miklós Zsolnay (1800–1880) in Pécs, Hungary, to produce stoneware and other ceramics in 1853. In 1863, his son, Vilmos Zsolnay (1828–1900) joined the company and became its manager and director after several years. He led the factory to worldwide recognition by demonstrating its innovative products at world fairs and international exhibitions, including the 1873 World Fair in Vienna, then at the 1878 World Fair in Paris, where Zsolnay received a Grand Prix. In 1893, Zsolnay introduced porcelain pieces made of eosin. Frost-resisting Zsolnay building decorations were used in numerous buildings specifically during the art nouveau movement. By 1914, Zsolnay was the largest company in Austro-Hungary. Due to historical and political reasons of the World wars and communism, the factory had some depressions, but afterwards the conditions improved. Beside the factory, there are two Zsolnay Museums in Pécs.

Eosin: Many Zsolnay ceramics are noted for the use of the eosin process that was introduced in 1893. The process results in an iridescence of the piece of work. Different eosin colours and processes were developed over time. The eosin-based iridescence became a favourite of art nouveau and Jugendstil artists. The secret eosin glaze renders porcelain to appear iridescent metallic, in different colours that change with the angle of reflection. Typical colours include shades of green, red, blue, and purple. You can enter the City Hall at the Széchenyi Square to watch the eosin clock oppositely the entrance, behind the reception desks. Souvenirs like eosin necklaces or porcelain art works can be bought in Zsolnay Porcelain shops one is near to the Széchényi Square, an other is in the Zsolnay Cultural Quarter which you can find during the free time of the excursion. (Be alerted that Zsolnay products are not cheap.)

Pyrogranite refers to a type of ornamental ceramics that were developed and produced by Zsolnay. Fired at high temperature, this durable material remains acid and frost-resistant making it suitable for use as roof tiles, indoor and outdoor decorative ceramics, and fireplaces. It can be seen in buildings such as Matthias Church, the Hungarian Parliament Building, the Museum of Applied Art, the Geological Institute, the Kőbánya Church, the Gellért Baths (all these buildings are in Budapest), the Post Office Palace, and County Hall in Pécs.

You should not ask for the recipes, they are strict secrets of the factory. You can visit two Zsolnay Museums, one is in the City Centre, which gives a broad picture on Zsolnay porcelain. The other one is located in the Zsolnay Cultural Quarter and bears the name the Gyugyi Collection, this one presents pieceworks. (Visiting a shop gives you a sense of ongoing products but be prepared for high prices.)

Zsolnay Museum in the City Center

Address: 4 Káptalan St. Opening hours: Tuesday-Sunday 10 a.m - 18 p.m. Closed on Mondays.

This exhibition, which displays a wide variety of pieces of the Zsolnay Ceramic Factory along with the history of the family, is in the oldest known dwelling-house of Pécs.

Figure 7: Zsolnay porcelain statue, photo by György Mánfai

Golden Age of Zsolnay – Gyugyi Collection in the Zsolnay Cultural Quarter

The pieces of the collection of Dr. László Gyugyi returned to their birthplace. The once world-famous factory of Vilmos Zsolnay today gives home to all these artworks as a renewed cultural part of Pécs. The museum concentrates on masterpieces, representing the golden age of the Zsolnays. Opening hours: Monday to Sunday: 10:00 am – 6:00 pm. On Sunday we will have a 1 hour free time in the Zsolnay Cultural Quarter around 15:00-16:00. For visiting this Zsolnay Museum, form a group of at least 10 people for 15 percent discount.

Full-price: HUF 1.600 Students / Teachers / Pensioners: HUF 1.000 University Pass card discount 15 percent.

THE MOSQUE OF PASHA JAKOVAI HASZAN

The mosque, which is the most intact Turkish mosque in Hungary, dates from the 16th century. It houses an exhibition of Turkish historical and artistic objects, and at the same time it is a Muslim place of worship.

Tel: +36 20 400 9301, Address: 2 Rákóczi street. Open: Tue-Sun: 10-18, Mon: closed. Last admission is at 17.30. Islamic Prayer time at 14.30 hours on Friday. Admission: It is free of charge for the purpose of praying.

GALLERY OF MODERN HUNGARIAN ART

Address: 5 Papnövelde Street. Opening hours: Tuesday-Sunday 10 a.m - 18 p.m. Closed on Mondays.

The new permanent exhibition of the Modern Hungarian Gallery presents the broadly interpreted 20th century Hungarian Art. It means that representatives of 19th century tendencies who survived themselves by previewing future, can be discovered in it, together with the characters of the end of the twentieth century who are already being considered as contemporaries now. The arrangement of the show basically follows the art historical chronological order, but sometimes some dilemmas overwrite it.

ETHNOGRAPHICAL MUSEUM

Address: 15 Rákóczi street. Tuesday - Saturday 10:00-16:00. Closed on Sundays and Mondays.

Showcases ethnic Hungarian, German and South Slav folk art in the region.

6 Lipót (Leopold) Fejér, a Fourier Analyst born in Pécs

Leopold Fejér was born in Pécs in a Jewish family as under the name Leopold Weiss, and changed his name to the Hungarian version Fejér around 1900.

Pólya writes about his personality:

"He had artistic tastes. He deeply loved music and was a good pianist. He liked a well-turned phrase. 'As to earning a living', he said, 'a professor's salary is a necessary, but not sufficient condition.' Once he was very angry with a colleague who happened to be a topologist, and explaining the case at length he wound up by declaring '... and what he is saying is a topological mapping of the truth'."

In the same article Pólya writes about Fejér's style of mathematics: "Fejér talked about a paper he was about to write up. 'When I write a paper,' he said, 'I have to re-derive for myself the rules of differentiation and sometimes even the commutative law of multiplication.' These words stuck in my memory and years later I came to think that they expressed an essential aspect of Fejér's mathematical talent; his love for the intuitively clear detail.

It is due to his care spent on the elaboration of the solution that Fejér's papers are very clearly written, and easy to read and most of his proofs appear very clear and simple. Yet only the very naive may think that it is easy to write a paper that is easy to read, or that it is a simple thing to point out a significant problem that is capable of a simple solution."

7 Cultural events in the city during the workshop

25th, 20:00. Concert in the Basilica. Haydn: Creation. Price: 2 900Ft. www.jegymester.hu and on the spot.

25th, Kodály Center. 18:30. **Kodály Zoltán: Székelyfonó**. Hungarian folk songs and dances. This program is our recommendation. We will have a list at the registration desk, on which we will match those who have a ticket. If interested, purchase tickets in advance at www.jegymester.hu.

Between 25-31: Fénycsapda, a silkpainter's exhibition.

27th, 18-19, „Nyáresti térzenék a Zsolnay negyedben" open air wind instrument orchestra, Zsolnay Quarter, E/78, free.

31st, 19-20. Carl Orff: Carmina Burana, open air Concert, Dom Square, free.

The sightseeing "train" leaves every hour during 10a.m. and 6p.m. See http://pecsikisvonat.hu/htmls

We recommend an adventure park for accompanying persons in good physical condition. http://www.mecsextrem.hu/

8 Directions

Buses will start from the University to the Headquarter at 8:30 in the morning of 25th, 26th, 29th, 30th of August to the Headquarter. There will be a bus after the welcome party back to Koch Dormitory. On 27th the bus leaves at 9:00 for the excursion.

Pécs has a fairly good system of public transportation by bus. Check the timetable and other details by clicking to http://mobilitas.biokom.hu/menetrend, however the site is not quite useful in English.

Bus number 32, 34Y, 35Y, or 36 take you from the train station and city center to the building of Academia Headquarter. Name of the Station in Hungarian is „MTA Székház". The ride takes 12 minutes.

Alternatively, you can take a taxi with Volán Taxi (+36-72-333-333). Most of the Taxis in Pécs are on acceptable price. The ride takes 5 minutes from the city center to the Academy's Headquarters and it costs about 1200 HUF (4 Euro).

Googlemaps site (https://www.maps.google.hu) also provides a reliable information; please do not forget to tick the "public transportation" or "pedestrian" option.

Sources: www.pecsimuzeumok.hu, www.pecsorokseg.hu/pecsorokseg_latogatokozpont, www.zsolnaynegyed.hu/lista/Zsolnay_negyed_gyugyi_gyujtemeny, www.wikipedia.com, maps.google.com.

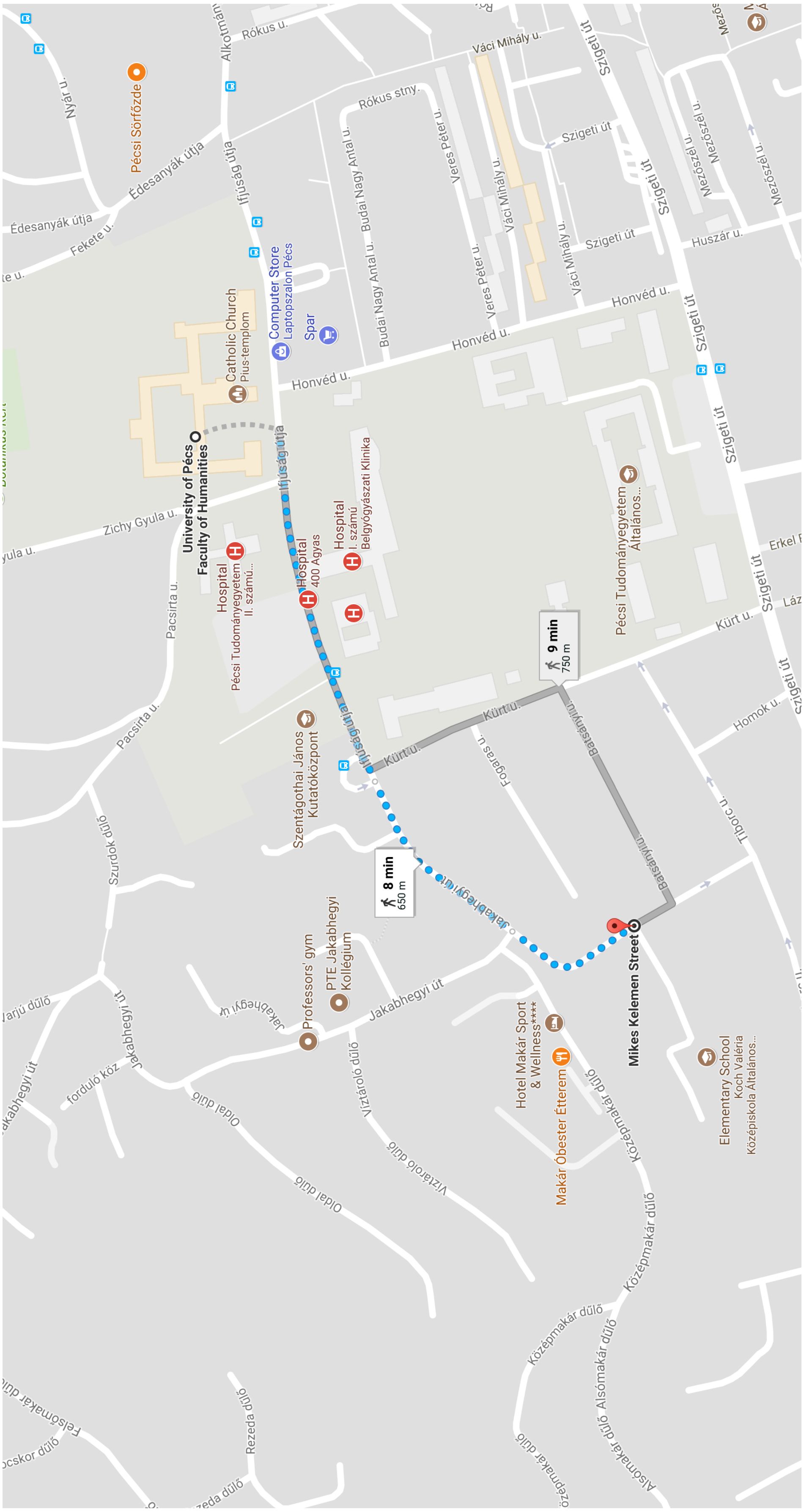

Figure 8: From Koch Dormitory (13 Mikes Kelemen street) to the University (6 Ifjúság street, venue of breakfasts for Koch guests and scientific program on August 28)

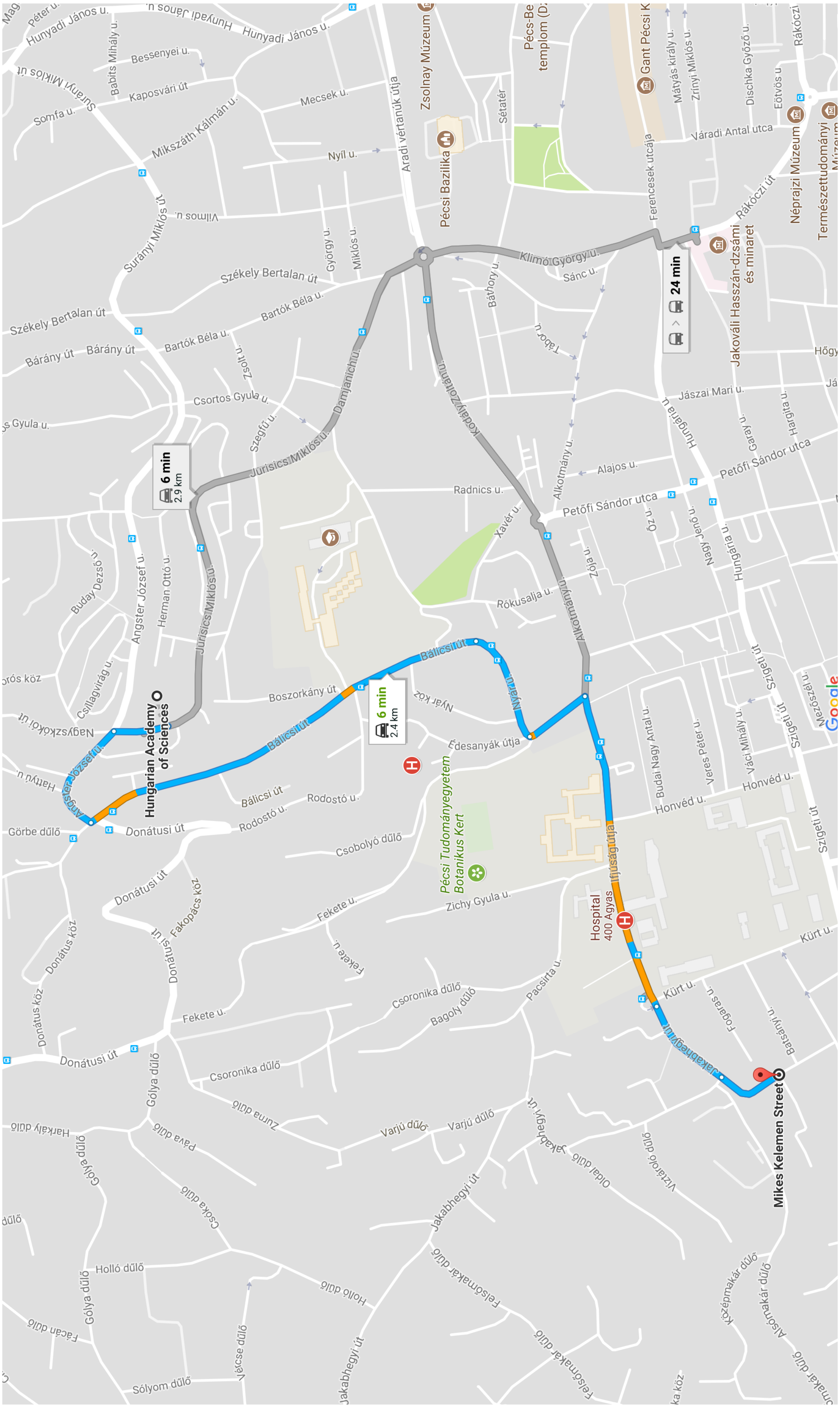

Figure 9: From the Headquarter of the Academy (44 Jurisics Miklós street) to Koch Dormitory (13 Mikes Kelemen street) by car

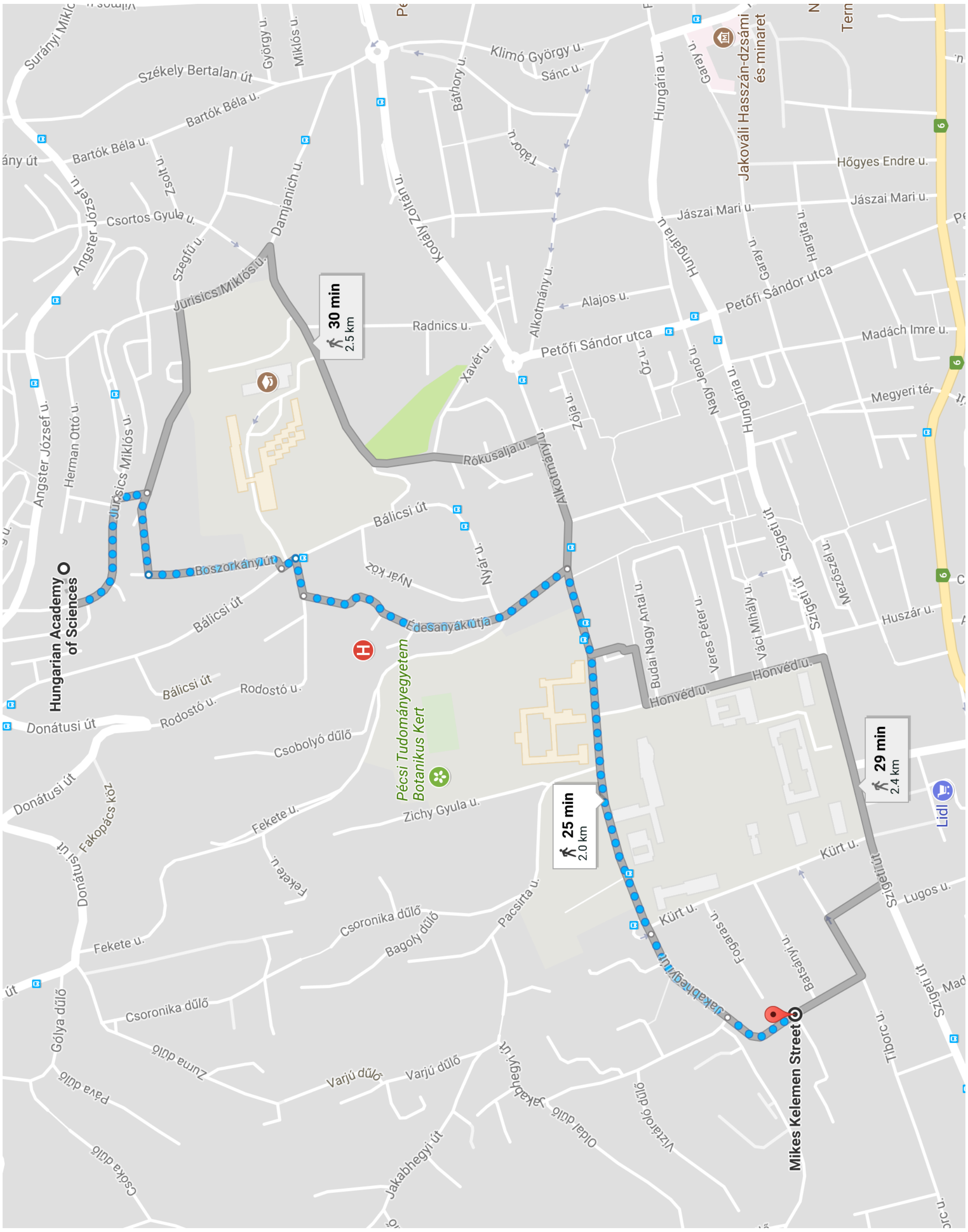

Figure 10: From the Headquarter of the Academy (44 Jurisics Miklós street) to Koch Dormitory (13 Mikes Kelemen street) on foot and University (6 Ifjúság street, with green letters: Pécsi Tudományegyetem Botanikus kert, entrance from the front, left to the box indicating "25 min")

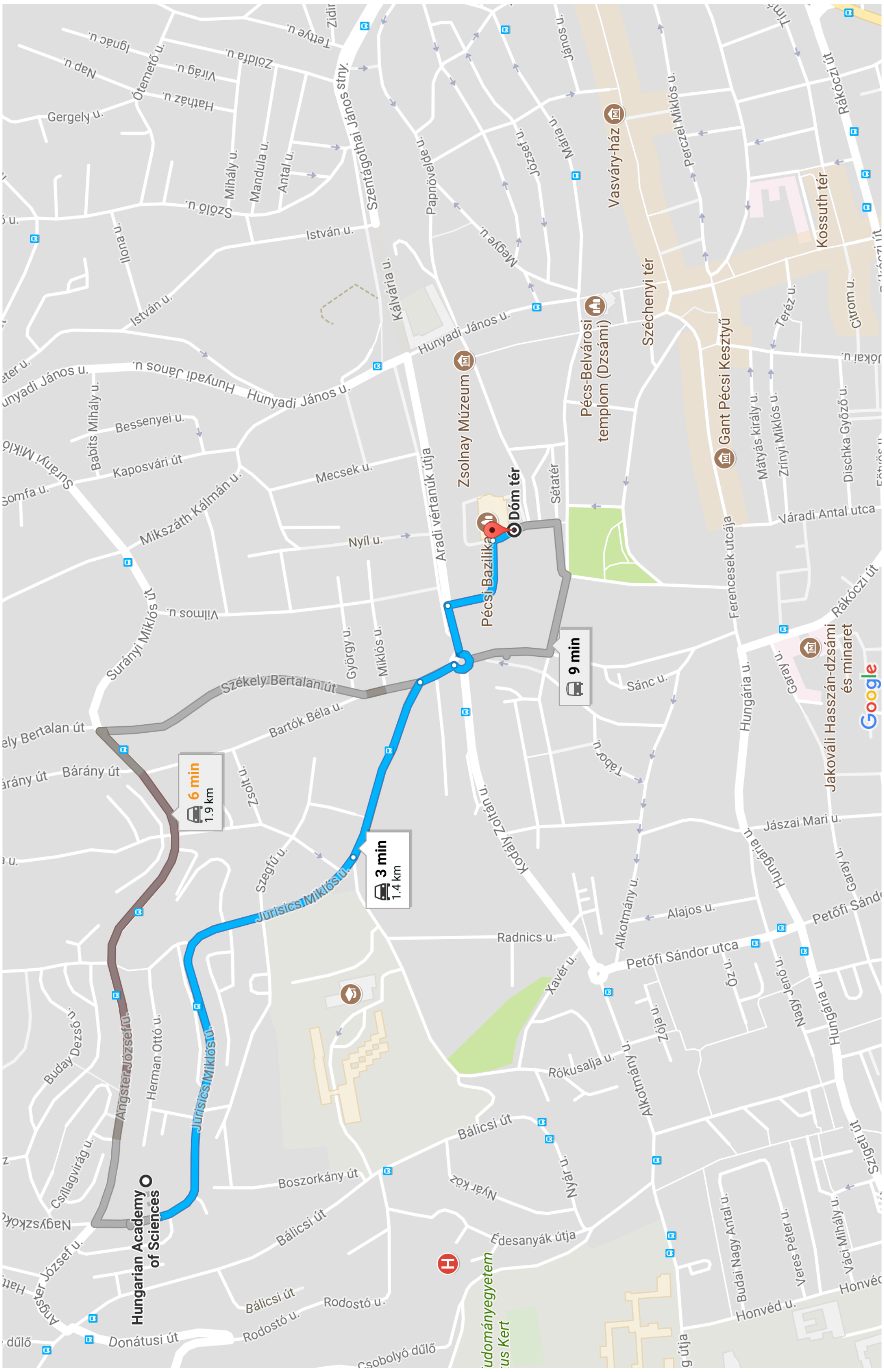

Figure 11: From the Headquarter of the Academy (44 Jurisics Miklós street) to the city center

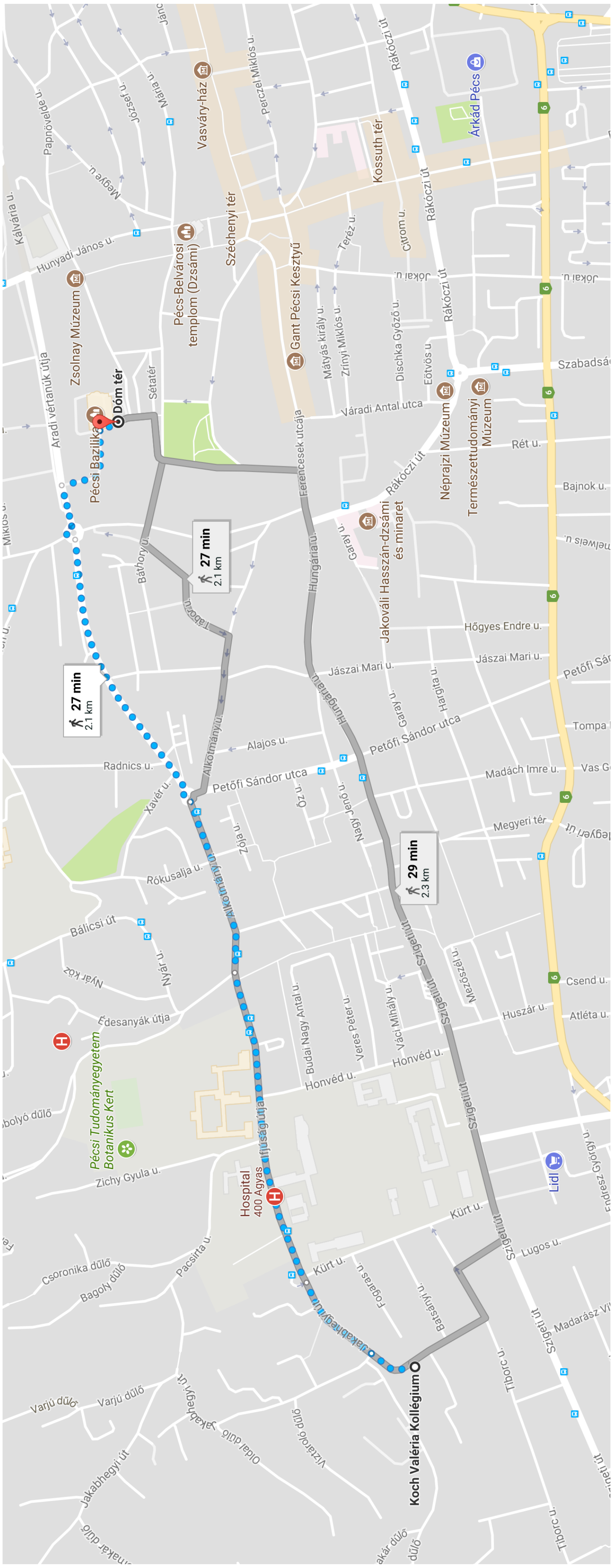

Figure 12: From Koch Dormitory (13 Mikes Kelemen street) to the city center