Lebesgue Functions of Rational interpolations of Non-band-limited Functions

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Sampling and interpolation

- For band-limited signals there is a classical result, the Whittaker-Kotelnikov-Shannon sampling theorem, which says that f can be reconstructed from samples at $n\pi/b$ by the formula $f(t) = \sum_{n=-\infty}^{\infty} f(n\pi/b) \frac{\sin b(t-n\pi/b)}{b(t-n\pi/b)}$.
- This holds for b-band-limited signals with finite energy, i.e. for functions $f \in L^2(\mathbb{R})$ whose Fourier transform has support in $[-b, b]$. The space of all such functions is the Paley-Wiener space $PW([-b, b])$.
- **Our purpose is to give similar reconstruction methods in case of non-band-limited signals.**

Sampling and interpolation

The Whittaker-Kotelnikov-Shannon sampling theorem follows from the properties of the Paley-Wiener space. The $PW[-b, b]$ is a reproducing kernel Hilbert space with reproducing kernel

$$k(t,u) = \begin{cases} \frac{\sin b(t-u)}{\pi(t-u)}, & t \neq u; \\ b/\pi, & t = u. \end{cases}$$

The function k has zeros at $t = m\pi/b$, $u = n\pi/b$, and the localized kernels

$$\sqrt{\pi/b}k_{n\pi/b}(t) = \sqrt{\pi/b}\frac{\sin b(t-n\pi/b)}{b(t-n\pi/b)}$$

form an orthonormal basis for $PW[-b, b]$. The sampling theorem for $J = [-b, b]$ is equivalent to the orthonormal expansion

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{\pi}{b}\langle f, k_{n\pi/b}\rangle k_{n\pi/b}(t) \qquad (t \in \mathbb{R}).$$

Sampling and interpolation

The steps of the proof of this theorem can't be extended if instead of J compact we consider $J = (0, \infty)$.

Remarks.

- $H^2(\mathbb{C}_+)$ is isomorphic with $\mathcal{F}^{-1}(L^2(0, +\infty))$
- Because of

$$f(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{f(t)}{t - z} dt = \int_{0}^{\infty} \widehat{f}(\xi) e^{2\pi i \xi z} d\xi, \qquad (z \in \mathbb{C}_+)$$

  we have the reproducing kernel $K(z, t) = \frac{1}{2\pi i(\bar{t} - z)}$ $(t \in \mathbb{R}, z \in \mathbb{C}_+)$.
- This has no zeros, consequently there are no nodes (t_n) for which the localized $K(t_n, t)$ would form an orthogonal basis for $H^2(\mathbb{C}_+)$.

New rational interpolations: T. Eisner, B. Király, M. Pap, Á. Pilgermájer

In the last years **new rational interpolations** has been investigated and lead to **new sampling and interpolation theorems.**
Idea: we approximate the Cauchy kernel K by a sequence of reproducing kernels K_N. Using the localized K_N-s on the zeros, we construct orthogonal and discrete orthogonal bases for some rational function spaces and in these spaces we give the analogue of the sampling theorem.
The construction of these operators is **based on the discrete orthogonality of the Malmquist-Takenaka systems**. Combining these interpolations one can give exact interpolation on the real line for a large class of rational functions among them for the Runge test function. The properties of the Lebesgue function of these rational interpolation operators were studied.

The Hardy space of the upper and lower half plane

Let $\mathbb{C}_+ = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$, $\mathbb{C}_- = \{z \in \mathbb{C} : \mathrm{Im}(z) < 0\}$. The set of holomorphic functions are denoted by $\mathrm{H}(\mathbb{C}_+)$, $\mathrm{H}(\mathbb{C}_-)$ and the corresponding Hardy spaces by

$$H^2(\mathbb{C}_+) = \left\{ h \in \mathrm{H}(\mathbb{C}_+) : sup \left\{ \int_{\mathbb{R}} |h(x+iy)|^2 \, dx : y > 0 \right\} < \infty \right\}.$$

$$H^2(\mathbb{C}_-) = \left\{ h \in \mathrm{H}(\mathbb{C}_-) : sup \left\{ \int_{\mathbb{R}} |h(x+iy)|^2 \, dx : y < 0 \right\} < \infty \right\}.$$

For each $f \in H^2(\mathbb{C}_+)$ there exists its non-tangential limit in $L^2(\mathbb{R})$. The Fourier transform of the boundary limit of $f \in H^2(\mathbb{C}_+)$ has support in $[0, \infty)$.
For each $f \in H^2(\mathbb{C}_-)$ there exists its non-tangential limit in $L^2(\mathbb{R})$. The Fourier transform of the boundary limit of $f \in H^2(\mathbb{C}_-)$ has support in $(-\infty, 0]$.

Malmquist-Takenaka systems for upper half plane

Let $\{\lambda_i\}_{i=1}^{\infty}$ an arbitrary sequence of complex numbers from the upper half plane $\mathbb{C}_+$, and let the **Malmquist-Takenaka system for the upper half plane** $\{\Psi_n\}_{n=1}^{\infty}$ defined by

$$\Psi_1(z) = \frac{\sqrt{\frac{\Im\lambda_1}{\pi}}}{z-\overline{\lambda_1}}, \quad \Psi_n = \frac{\sqrt{\frac{\Im\lambda_n}{\pi}}}{z-\overline{\lambda_n}} \prod_{k=1}^{n-1} \frac{z-\lambda_k}{z-\overline{\lambda_k}}, \quad (n = 2, 3, \ldots).$$

- This is a system of rational functions associated with the set of poles $\{\overline{\lambda_i}\}_{i=1}^{\infty}$ in the lower half-plane.

Malmquist-Takenaka systems for upper half plane

- The system of functions $\{\Psi_n\}_{n=1}^{\infty}$ is orthonormal on the entire axis $-\infty < x < +\infty$ in the following sense

$$\int_{-\infty}^{+\infty} \Psi_n(x)\overline{\Psi_m(x)}dx = \delta_{mn}.$$

- Moreover, if we have the following non-Blaschke condition for the upper half plane

$$\sum_{k=1}^{\infty} \frac{\Im\lambda_k}{1+|\lambda_k|^2} = \infty$$

then $\{\Psi_n\}_{n=1}^{\infty}$ is a complete orthonormal system for $H^2(\mathbb{C}_+)$.

The kernel function to M-T system and its partial sums

- The kernel function

$$K(z,\xi) = \sum_{k=1}^{\infty} \Psi_k(z)\overline{\Psi_k(\xi)} = \frac{1}{2i\pi(\overline{\xi} - z)}$$

- Blaschke like products: $\widetilde{B}_N(z) = \prod_{k=1}^{N} \frac{z-\lambda_k}{z-\overline{\lambda}_k}\eta_k$, where $N > 0, \eta_k = \frac{|1+\lambda_k^2|}{1+\lambda_k^2}$, if $\lambda_k \neq i$, and $\eta_k = 1$ if $\lambda_k = i$
- For arbitrary values of the variables $z \neq \overline{\xi}$ and for any N, $1 \leq N < \infty$, the analogue of the Darboux-Christoffel formula for the upper half plane

$$K_N(z,\xi) = \sum_{k=1}^{N} \Psi_k(z)\overline{\Psi_k(\xi)} = \frac{1 - \overline{\widetilde{B}_N(\xi)}\widetilde{B}_N(z)}{2i\pi(\overline{\xi} - z)}$$

Extended M-T system to the lower half plane

The extended system $\mathbf{\Psi}_N$ for negative indexes is given by:

$$\widetilde{B}_{\overline{N}}(z) = \prod_{k=1}^{N} \frac{z - \overline{\lambda_k}}{(z - \lambda_k)\eta_k},$$

$$\mathbf{\Psi}_{\overline{N}} = \{\Psi_{-n} = \widetilde{B}_{\overline{N}}\Psi_n, n = 1, 2, \ldots, N\}.$$

Discrete orthogonality

For $a_k = \frac{i-\lambda_k}{i+\lambda_k}$ let us consider the equation

$$\frac{z-a_1}{1-\overline{a_1}z} \cdot \frac{z-a_2}{1-\overline{a_2}z} \cdot \ldots \cdot \frac{z-a_N}{1-\overline{a_N}z} = 1$$

has N different solutions and they can be written as $z_k := e^{i\tau_k}$ Let denote by $t_k = \tan\frac{\tau_k}{2}$, where τ_k, $k = 1, \ldots, N$ i.e. $z_k = e^{i\tau_k} = \frac{i-t_k}{i+t_k}$, $k = 1, \ldots, N$. Let us introduce the following set of nodes on the real line

$$\mathbb{R}_N = \{t_k : \ k = 1, \ldots, N\}.$$

Discrete orthogonality

Furthermore, from the definition of z_k and t_k, one can get

$$\overline{\widetilde{B}_N(t_\ell)}\widetilde{B}_N(t_k) = \overline{\widetilde{B}_{\overline{N}}(t_\ell)}\widetilde{B}_{\overline{N}}(t_k) = B_N(z_k)\overline{B_N(z_\ell)} = 1. \qquad (1)$$

Suppose that every node is finite. Using the points of discretization one can get the formula of the localized reproducing kernels

$$K_N(t, t_k) = \begin{cases} \frac{1-\widetilde{B}_N(t)}{2\pi i(t_k - t)} & t \neq t_k, \\ \sum_{m=1}^{N} \frac{\Im\lambda_m}{\pi\left|t_k - \overline{\lambda_m}\right|^2} & t = t_k. \end{cases} \qquad (2)$$

$$K_{\overline{N}}(t, t_k) = \begin{cases} \frac{\widetilde{B}_{\overline{N}}(t) - 1}{2\pi i(t_k - t)} & t \neq t_k, \\ \sum_{m=1}^{N} \frac{\Im\lambda_m}{\pi|t_k - \lambda_m|^2} & t = t_k. \end{cases} \qquad (3)$$

Discrete orthogonality

Let define the following weight function:

$$\frac{1}{\widetilde{\rho}_N(t)} := K_N(t,t) = \sum_{k=1}^{N} \frac{\Im \lambda_k}{\pi |t - \overline{\lambda_k}|^2} \quad (t \in \mathbb{R}),$$

and the following discrete scalar product:

$$\langle F, G \rangle_N = \sum_{t \in \mathbb{R}_N} F(t)\overline{G(t)}\widetilde{\rho}_N(t).$$

Discrete orthogonality, Eisner-P, JFAA, 2014

Theorem

The finite collection of $\Psi_n (1 \leq n \leq N)$ *forms a discrete orthonormal system with respect to the scalar product*

$$\langle F, G \rangle_N = \sum_{t \in \mathbb{R}_N} F(t)\overline{G(t)}\widetilde{\rho}_N(t),$$

namely

$$\langle \Psi_n, \Psi_m \rangle_N = \delta_{mn} \quad (1 \leq m, n \leq N).$$

Similarly the finite collection of $\Psi_n (n = -N, -N+1, \dots, -1)$ *forms a discrete orthonormal system with respect to the scalar product* $\langle ., . \rangle_N$, *i.e.,*

$$\langle \Psi_n, \Psi_m \rangle_{\overline{N}} = \delta_{mn} \quad (-N \leq m, n \leq -1).$$

Projection operators on special spaces of rational functions

Let denote by $\mathcal{P}_k$ the space of polynomials of degree at most k, $\eta(z) = \prod_{n=1}^{N}(z - \overline{\lambda_n})$, $\omega(z) = \prod_{n=1}^{N}(z - \lambda_n)$ and set

$$\mathcal{R}_N := \left\{ \frac{p}{\eta} : \; p \in \mathcal{P}_{N-1} \right\}$$

$$\mathcal{R}_{\overline{N}} := \left\{ \frac{p}{\omega} : \; p \in \mathcal{P}_{N-1} \right\}$$

$$\mathcal{R}_{N,\overline{N}} := \left\{ \frac{p}{\eta\omega} : \; p \in \mathcal{P}_{2N-1} \right\}$$

It is clear that $\mathcal{R}_{N,\overline{N}} = \mathcal{R}_N \oplus \mathcal{R}_{\overline{N}}$, i.e., they are orthogonal complement in $L^2(\mathbb{R})$.

Projection on $\mathcal{R}_N$ and on $\mathcal{R}_{\overline{N}}$

$$\mathcal{R}_N = \mathrm{span}\{\Psi_\ell,\ \ell = 1, \dots, N\}, \quad \mathcal{R}_{\overline{N}} = \mathrm{span}\{\Psi_\ell,\ \ell = -1, \dots, -N\}.$$

Let us consider the orthogonal projection operator of an arbitrary function $f \in H^2(\mathbb{C}_+)$ on the subspace $\mathcal{R}_N$ given by

$$P_N f(z) = \sum_{k=1}^{N} \langle f, \Psi_k \rangle \Psi_k(z).$$

Analogously the orthogonal projection operator of an arbitrary function $f \in H^2(\mathbb{C}_-)$ on the subspace $\mathcal{R}_{\overline{N}}$ is

$$P_{\overline{N}} f(z) = \sum_{k=-N}^{-1} \langle f, \Psi_k \rangle \Psi_k(z).$$

The pointwise convergence

Theorem

Let suppose that the non-Blaschke condition is satisfied for the parameters λ_n. Then for any $f \in H^2(\mathbb{C}_+)$ and any $z \in \mathbb{C}_+$ we have $P_N f(z) \to f(z)$, and for any $f \in H^2(\mathbb{C}_-)$ and any $z \in \mathbb{C}_-$ we have $P_{\overline{N}} f(z) \to f(z)$.

From the proof it follows that $P_N f \to f$ uniformly on every compact subset of the upper half plane and $P_{\overline{N}} f \to f$ uniformly on every compact subsets of the lower half plane.
We are also interested in the behaviour of P_N and $P_{\overline{N}}$ on the real line.

Convergence on the real line

Theorem

If $f \in H^2(\mathbb{C}_+)$ *has a partial fraction decomposition* $f(z) = \sum_{\ell=1}^{m} \frac{c_\ell}{z-\overline{\gamma_\ell}}, \quad \gamma_\ell \in \mathbb{C}_+$, *then* $|f(t) - P_N f(t)| \to 0$ *uniformly on* $\mathbb{R}$ *and* $\lim\limits_{N\to 0} \max_{t\in\mathbb{R}}(1+t^2)|f(t) - P_N f(t)|^2 \to 0$.

Analogously, if $f \in H^2(\mathbb{C}_-)$ *has a partial fraction decomposition* $f(z) = \sum_{\ell=1}^{m} \frac{c_\ell}{z-\gamma_\ell}, \quad \gamma_\ell \in \mathbb{C}_+$, *then* $|f(t) - P_{\overline{N}} f(t)| \to 0$ *uniformly on* $\mathbb{R}$ *and* $\lim\limits_{N\to 0} \max_{t\in\mathbb{R}}(1+t^2)|f(t) - P_{\overline{N}} f(t)|^2 \to 0$.

Properties of the projection operators

Corollary

For every $f \in \mathcal{R}_N$ the corresponding discrete and continuous Malmquist-Takenaka coefficients are equal, i.e.

$$\langle f, \Psi_k \rangle = \langle f, \Psi_k \rangle_N, \ (1 \leq k \leq N), \tag{4}$$

and

$$P_N f(z) = \langle f(.), K_N(., z) \rangle = \langle f(.), K_N(., z) \rangle_N = f(z) \quad (z \in \mathbb{C}_+).$$

Similarly, for every $f \in \mathcal{R}_{\overline{N}}$

$$\langle f, \Psi_k \rangle = \langle f, \Psi_k \rangle_N, \ (-N \leq k \leq -1),$$

$$P_{\overline{N}} f(z) = \langle f(.), K_{\overline{N}}(., z) \rangle = \langle f(.), K_{\overline{N}}(., z) \rangle_N = f(z) \quad (z \in \mathbb{C}_-). \tag{5}$$

Rational interpolation with nodes on the real line

Let $\mathbb{R}_N$ to be the set of nodes. Consider the following interpolation operators:

$$\mathcal{L}_N f := \sum_{t \in \mathbb{R}_N} \frac{K_N(.,t)}{K_N(t,t)} f(t),$$

where f is in $\mathbb{A}(\mathbb{C}_+)$.
Analogously, for the lower half plane algebra of analytic functions $\mathbb{A}(\mathbb{C}_-)$ consider the following interpolation operators:

$$\mathcal{L}_{\overline{N}} f := \sum_{t \in \mathbb{R}_N} \frac{K_{\overline{N}}(.,t)}{K_{\overline{N}}(t,t)} f(t),$$

where $f \in \mathbb{A}(\mathbb{C}_-)$.

Rational interpolation with nodes on the real line

Let us denote by

$$\ell_{N,t}(\omega) := \frac{K_N(\omega,t)}{K_N(t,t)},\ \ell_{\overline{N},t}(\omega) := \frac{K_{\overline{N}}(\omega,t)}{K_{\overline{N}}(t,t)},\quad (t\in\mathbb{R}_N, \omega\in\mathbb{R}).$$

From the definition of $\mathbb{R}_N$, K_N, $K_{\overline{N}}$ and (1) it follows that for $1 \leq k, \ell \leq N$, one has:

$$\ell_{N,t_k}(t_\ell) = \frac{K_N(t_\ell,t_k)}{K_N(t_k,t_k)} = \delta_{k\ell},\ \ell_{\overline{N},t_k}(t_\ell) = \frac{K_{\overline{N}}(t_\ell,t_k)}{K_{\overline{N}}(t_k,t_k)} = \delta_{k\ell}$$

i.e., $\{\ell_{N,t},\ t\in\mathbb{R}_N\}$ are the Lagrange functions corresponding to the system $\{\Psi_\ell,\ \ell=1,\ldots,N\}$, and $\{\ell_{\overline{N},t},\ t\in\mathbb{R}_N\}$ are the Lagrange functions corresponding to the system $\{\Psi_\ell,\ \ell=-N,\ldots,-1\}$. **This implies that $\mathcal{L}_N f$ and $\mathcal{L}_{\overline{N}} f$ interpolate f at the points of $\mathbb{R}_N$.**

Rational interpolation with nodes on the real line

Using the reproducing property of K_N and $K_{\overline{N}}$ it can be proved that $\ell_{N,t}(\omega)$, $(t \in \mathbb{R}_N)$ form an orthogonal basis in $\mathcal{R}_N$ and $\ell_{\overline{N},t}(\omega)$, $(t \in \mathbb{R}_N)$ form an orthogonal basis in $\mathcal{R}_{\overline{N}}$.
The interpolation operators can be expressed also using the discrete scalar product as:

$$\mathcal{L}_N f(z) = \langle f, K_N(.,z)\rangle_N \quad (f \in \mathbb{A}(\mathbb{C}_+),\ z \in \mathbb{C}_+),$$

$$\mathcal{L}_{\overline{N}} f(z) = \langle f, K_{\overline{N}}(.,z)\rangle_N \quad (f \in \mathbb{A}(\mathbb{C}_-),\ z \in \mathbb{C}_-).$$

From (4) and (5) it follows that these operators are exact on $\mathcal{R}_N$ and $\mathcal{R}_{\overline{N}}$ respectively, i.e.,

$$\mathcal{L}_N f = P_N f = f,\ f \in \mathcal{R}_N, \quad \mathcal{L}_{\overline{N}} f = P_{\overline{N}} f = f,\ f \in \mathcal{R}_{\overline{N}}.$$

Properties of the interpolation operators

As a consequence of the previous property we can propose a new exact interpolation scheme for those functions which belong to $\mathcal{R}_{N,\overline{N}}$. Let $f \in \mathcal{R}_{N,\overline{N}}$, then $f = f_1 + f_2$, where $f_1 \in \mathcal{R}_N$ and $f_2 \in \mathcal{R}_{\overline{N}}$ and let define $L_N f = \mathcal{L}_N f_1 + \mathcal{L}_{\overline{N}} f_2$. Then for every $f \in \mathcal{R}_{N,\overline{N}}$

$$L_N f = \mathcal{L}_N f_1 + \mathcal{L}_{\overline{N}} f_2 = f_1 + f_2 = f.$$

If we choose $\lambda_1 = i$, then the Runge's test function belongs to $\mathcal{R}_{N,\overline{N}}$. Indeed

$$f(z) = \frac{1}{z^2+1} = \frac{1}{2i(z-i)} - \frac{1}{2i(z+i)}.$$

Taking $f_1 = \frac{-1}{2i(z+i)} \in \mathcal{R}_N$ and $f_2 = \frac{1}{2i(z-i)} \in \mathcal{R}_{\overline{N}}$ we obtain the following exact interpolation for the Runge's function:

$$L_N f = \mathcal{L}_N f_1 + \mathcal{L}_{\overline{N}} f_2 = f_1 + f_2 = f.$$

Convergence Properties of $\mathcal{L}_N f$

Theorem

Let $\lambda_1 = i, \lambda_k \in \mathbb{C}_+$ *such that*

$$\sum_{k=1}^{\infty} \frac{\Im \lambda_k}{1 + |\lambda_k|^2} = \infty.$$

If $f \in \mathbb{A}(\mathbb{C}_+)$ *is uniformly continuous on* $\overline{\mathbb{C}_+}$ *such that*

$$\lim_{N\to\infty} \max_{t\in\mathbb{R}} (1 + t^2)|f(t) - P_N f(t)|^2 = 0,$$

then the interpolation operator $\mathcal{L}_N f := \sum_{t\in\mathbb{R}_N} \frac{K_N(.,t)}{K_N(t,t)} f(t)$ *converges to* f *in norm, i.e.,* $\lim_{N\to\infty} \|f - \mathcal{L}_N f\|_2 = 0.$

Very similar result holds for the lower half plane.

The Lebesque function of the interpolation operator

The Lebesgue function associated to the interpolation problem is

$$\Lambda_N(t) = \sum_{t_k \in \mathbb{R}_N} |\ell_{N,t_k}(t)| = \sum_{t_k \in \mathbb{R}_N} \left| \frac{K_N(t, t_k)}{K_N(t_k, t_k)} \right| .$$

The Lebesgue constant is the maximum value of the Lebesgue function $\Gamma_N = \max_{t \in \mathbb{R}} \Lambda_N(t)$. Figures
At first we get for the associated Lagrange functions $\ell_{N,t_k}(t)$ of our rational interpolation, or equivalently the corresponding localized reproducing kernels

Theorem

For every $t_k \in \mathbb{R}_N$ the function $\ell_{N,t_k}(t) = \frac{K_N(t,t_k)}{K_N(t_k,t_k)}$ are continuous on $\mathbb{R}$ and they tend to 0 if $|t| \to \infty$.

The Lebesque function of the interpolation operator

We obtained theoretical bounds for Γ_∞^N, and we made numerical experiments.

$$\Lambda_N(t_\ell) = \sum_{t_k \in \mathbb{R}_N} \left| \frac{K_N(t_\ell, t_k)}{K_N(t_k, t_k)} \right| = 1, \quad t_\ell \in \mathbb{R}_N.$$

Secondly, $\Lambda_N(t)$ is continuous on $\mathbb{R}$ and has limit 0 if $|t| \to \infty$. Moreover, for every $\epsilon > 0$ there exists $\delta_\ell > 0$ such that, if $t \in (t_\ell - \delta_\ell, t_\ell + \delta_\ell)$ we have $|\Lambda(t) - 1| < \epsilon$, which implies that in a neighbourhood of every $t_\ell \in \mathbb{R}_N$ the function Λ_N is bounded by $1 + \epsilon$.

If $t \in [-P, P] \setminus [\cup_{\ell=1}^N (t_\ell - \delta_\ell, t_\ell + \delta_\ell)]$, then $\frac{1}{|t-t_k|} < \max_{\ell=1,\ldots,N} \frac{1}{\delta_\ell} =: M_N$, and

$$\Lambda_N(t) \leq \frac{M_N}{\pi} \sum_{k=1}^N \frac{1}{K_N(t_k, t_k)} \leq M_N(1 + P^2).$$

Referencelist

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Thank you for your attention!

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Figures of the Lebesgue function ($N = 81$) and its terms ($N = 81, t_k = 1, 40, 81$) Go Back